Hidden Competence: Women's Mathematical Participation in Public and Private Classroom Spaces

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This paper reports on gender and participation in one inquiry-oriented undergraduate mathematics course. While there is evidence that inquiry can better support all learners, especially women, less is known about the distribution of participation in inquiry classrooms. Here we focus on how men and women participated in both public spaces (in plenary with the whole class) and private spaces (with their group members). We found that while many women provided a large number of high-level "why" contributions in their groups, this did not necessary translate to participation in plenary discussions. We thus argue that women's competence was hidden to the whole class, which contributes to the systematic marginalization of women in mathematics.

Keywords: equity, gender, discourse, classroom participation

Introduction

Mathematics has a problem with patriarchy. From a very young age, girls are confronted with problematic narratives about who can and cannot do mathematics (e.g., Beilock, Gunderson, Ramirez, & Levine, 2010; Boaler, 2002; Robinson-Cimpian, Lubienski, Ganley, & Copur-Gencturk, 2014). Even young women with high levels of mathematics achievement often choose to leave the field, because these narratives create conflicts for women who have to reconcile their identities as mathematicians with their identities as women (e.g., Mendick, 2005; Walkerdine, 1998). This trend continues at the postsecondary level, where an uninviting weed-out culture further results in talented women opting out of mathematics (National Science Board, 2018; Seymour & Hewitt, 1997).

Despite this bleak situation, there is evidence that inquiry-based learning environments can help all students, and especially women, persist and be more successful at mathematics (Laursen, Hassi, Kogan, & Weston, 2014). Broadly defined, an inquiry-based classroom is one in which students collaboratively develop mathematical ideas though engaging in a series of classroom activities that require them to explore mathematics through problem solving and argumentation (Laursen & Rasmussen, in press; Mathematical Association of America, 2018). Such classrooms are grounded in a situated perspective of learning, which stipulates that learning results through participation in social practices (e.g., Lave, 1996). However, less is known about how such courses support particular forms of participation that are consequential to learning and persistence. Do such courses provide equitable participation opportunities for men and women? Do women get to make higher-level contributions, or just provide answers? Do such courses provide opportunities for women to be positioned as competent knowers and doers of mathematics?

To explore these questions, we studied participation in a single inquiry-based undergraduate geometry course. Our overall goal was to better understand when and how inquiry-based learning environments are most productive for women, and under what circumstances they may be less productive. Although not the focus of our study, we suspect that our results may also be relevant for students from other groups minoritized in mathematics. To capture the complexity of participation in various classroom spaces (e.g., group talk vs. plenary discussion), we used a total of five video cameras: four focused on groups at their tables, and one to capture the class as a whole. Because all cameras continued to collect data during plenary discussions, we were also able to investigate the private "side talk" that took place during plenary discussions.

This paper contributes to research on gender in postsecondary mathematics learning by drawing attention to everyday classroom interactions. This provides insight to how larger systemic issues (e.g., the climate for mathematics majors; Lubienski & Ganley, 2017) play out at the classroom level. We address two questions: (1) how were participation opportunities distributed across women and men in different classroom spaces? And (2) what opportunities did women have to be positioned as competent mathematicians?

We found that many women participated with numerous high-level contributions in small groups. These same women rarely, if ever, participated publicly during plenary discussions. However, we did find evidence that these women continued to participate at their tables in side talk. During the plenary discussions, there were no explicitly observable efforts from the instructor to include women. Moreover, our analyses revealed a number of overtly sexist interactions and microaggressions during small group work. Thus, while women did contribute meaningful mathematics through their participation in small groups, these contributions were not made public.

Conceptual Framework

Research shows that discourse can support students in learning mathematics (Michaels, O'Connor, & Resnick, 2008; National Council of Teachers of Mathematics, 2014; Sfard & Kieran, 2001). In many classrooms, plenary (whole-class) discussions are a primary site for students to participate in mathematical discourse. Discourse also occurs in less public spaces, such as small groups. We consider small groups as a semi-private space: although discourse in small groups might be audible to classmates nearby or a teacher circulating the room, most of the discussion will be confined to members of the small group. Further, discourse can happen in "side talk" between students. For example, students might whisper mathematical ideas or questions to each other in private, electing not to make this discourse publicly available. Such side talk can be productive, helping students deepen their mathematical understandings (Houssart, 2001).

Across public and private spaces, students vie for opportunities to participate on these "conversational floors" (Engle, Langer-Osuna, & McKinney de Royston, 2014). This is one place where discourse and learning converge with issues of equity and inequity. In this paper, we focus on participatory equity, which concerns the fair distribution of both participation and opportunities to participate in core aspects of the learning process, including classroom discourse (Shah & Lewis, in press). Research shows that participatory inequities abound in classrooms, especially along gender and racial lines (Esmonde & Langer-Osuna, 2013; McAfee, 2014; Sadker, Sadker, & Zittleman, 2009). In this article, we assume that a baseline for participatory equity is that students from marginalized groups in mathematics should *at least* participate at rates proportional to their demographic representation in a classroom. As we explain later, part of our approach here utilizes equity analytics, a methodology designed to illuminate participatory equity (see Reinholz & Shah, 2018). The ultimate purpose of equity analytics is to generate quantitative patterns of participation that can illuminate potential implicit biases in teaching, for use in professional development (Herbel-Eisenmann & Shah, in press; Reinholz, Bradfield, & Apkarian, 2019). The EQUIP web app (https://www.equip.ninja) is a free, customizable tool that facilitates others to use the equity analytics approach. In this study, we focus on equity analytics as a research tool, not for professional development.

Public Classrooms Spaces as Critical Sites for Positioning Students

Participation patterns should be monitored wherever students engage in classroom discourse. However, participation in plenary discussions is especially crucial because this is where students become publicly recognized as mathematically competent. This matters for at least two reasons. First, being publicly positioned as mathematically competent can help build students' mathematics identities, or the extent to which they see themselves as capable of learning and doing mathematics (Boaler & Greeno, 2000; Martin, 2000). Learning and identity formation are closely related processes, and for some students building a domain identity can facilitate their learning of a discipline (Hand & Gresalfi, 2015; Nasir, 2002). Pedagogical approaches like Complex Instruction recognize the public assignation of domain competence as a way to also elevate the status of marginalized students (Cohen & Lotan, 1997; Featherstone et al., 2014). Complex Instruction is a particular set of techniques grounded in sociological theory, designed to promote equitable groupwork, and the efficacy of these techniques for improving equity is well-documented (e.g., Boaler, 2006).

Second, for students from marginalized groups, the public positioning of competence is also a matter of cultural representation. For example, imagine a scenario where Latinx students—a group often falsely positioned as mathematically deficient (Gutiérrez, 2002; Shah, 2017)—are high performers in a mathematics class but rarely participate publicly. While they may find personal success, it does little to shape their non-Latinx classmates' perceptions of Latinx people as a group in mathematics. Changing these broader cultural discourses is difficult, but the public showcase of competence can make incremental differences over time.

Of course, more private spaces for classroom discourse matter as well. Small-group work can have real benefits for student learning (Hmelo-Silver & Barrows, 2008; Webb, 1991). Small-group work can be perceived as a more private "safe space" for initial idea generation before public presentation, functioning as a springboard for plenary discussion. However, there is no guarantee that small-group participation will translate to public participation. Thus, it is important we research how participation in more private spaces compares to participation in public spaces.

Women's Experiences in Undergraduate Mathematics Education

Opportunities to be publicly positioned as mathematically competent are especially important for women. Women's mathematical competence should not be disputed. They are obviously capable of excelling in mathematics. And yet, sexist narratives about women's lack of innate mathematical ability persist in the field (Boaler, 1997; Leyva, 2017; Mendick, 2006;

Walshaw, 2001). Research shows that such false narratives marginalize girls in mathematics as early as elementary school (Beilock et al., 2010; Robinson-Cimpian et al., 2014), and that this process continues into both the secondary level (Brown & Leaper, 2010) and post-secondary mathematics education (Seymour & Hewitt, 1997).

Much of the research on the marginalization of women in mathematics comes from the United States, Europe, and Australia, although research does also document significant marginalization elsewhere, such as across Africa (see Leder, 2015; Masanja, 2010). For this reason, it is clear that patriarchy in mathematics must be recognized as a global issue. Even in countries considered more gender equitable, such as Sweden, women are still underrepresented in higher-level mathematics arenas (Brandell, 2008).

Research shows that mathematics self-concept, or one's perception of their own abilities in mathematics, is a predictor of persistence in mathematics (e.g., Ellis, Fosdick, & Rasmussen, 2016; Sax et al, 2015). Yet, as a result of problematic narratives, even highly competent women in undergraduate mathematics courses may struggle to recognize their own abilities (Rodd & Bartholomew, 2006). What is encouraging, though, is that research also shows that these beliefs are malleable, and that they can be impacted through targeted interventions (Gaspard et al., 2015). Still, we want to be clear that we do not interpret the results of these studies to mean that we need to "fix" women in mathematics by making them more confident; rather, we need to fix the systemic patriarchy that erodes women's confidence.

Much of the research on gender in mathematics education—including many of the above studies—focuses on women's experiences along the "pipeline," as they traverse potentially hostile "climates" in post-secondary mathematics departments. This focus is summarized in a recent synthesis of research on gender in mathematics education (see Lubienski & Ganley, 2017). In part, this motivated our focus in this article on women's experiences in everyday teaching and learning interactions, which we now consider.

Mathematics classrooms are spaces where power is negotiated and students belonging to certain social groups must resist hierarchies that position them as inferior (Esmonde & Langer-Osuna, 2013; Mendick, 2006; Walshaw, 2001). For women, this power typically manifests in sexism and sometimes outright misogyny. At times, sexism can be explicit and flagrant (e.g., a man telling a woman that her ideas don't matter because he thinks women can't do mathematics), but sexism is often also subtle. Social psychologists use the term "microaggressions" to refer to "brief, everyday exchanges that send denigrating messages to certain individuals because of their group membership" (Sue, 2010, p. xvi; cf. Pierce, 1970). Examples of gender microaggressions include people in a group taking up a man's ideas while ignoring a woman's ideas, or a woman at her workplace constantly receiving comments about her physical appearance rather than the quality of her work. Further, stereotype threat research shows that sexist narratives can have an imperceptible but potent effect on women even when not explicitly invoked (Spencer, Steele, & Quinn, 1999). That is, even when sexism is not deployed in social interaction, sexism "in the air" can have detrimental effects on women's experiences in academic settings.

To be clear, all men in mathematics education are not sexist against women or people who do not identify with the traditional gender binary. Relatedly, all women in mathematics education do not experience sexism in the same ways. Following Boaler (2002), we eschew essentialist and monolithic accounts of women's experiences in mathematics. What we point to here, though, is that, historically speaking, mathematics education has been organized by a patriarchal, exclusionary culture, and that women's participation in classroom discourse must be analyzed as situated within sexist discourses of masculinity (Leyva, 2017; Mendick, 2006). Not surprisingly, dynamics around gender in undergraduate mathematics education reflect persistent sexism in society writ large. Our goal is to analyze gender patterns in classroom discourse to document both the barriers women experience in mathematics classrooms and the mathematical competence they possess. The latter provide a counternarrative to the dominant discourse about women in mathematics, such that their competence does not remain a hidden competence.

Method

Participants and Context

The study took place in an undergraduate geometry course co-taught by two mathematics educators at a large, PhD-granting Hispanic Serving Institution (HSI)[1]. Four small groups of four students were chosen to be observed over the course of the semester, which resulted in 16 out of 29 students in the class participating in this study. These groups consisted of three gender-balanced groups (two women and two men), and one group with three women and one man.

Of these 16 students, 14 were undergraduates: ten were seeking a Bachelor of Arts in mathematics, in preparation for earning a teaching credential in secondary school mathematics, two were studying mathematics (not seeking a credential), one was studying chemistry, and one was studying engineering. The other two students were pursuing a Master of Arts in Mathematics Education. The demographics of the students (as identified by a course instructor) are given in Table 1. We focus on teacher identification, rather than student self-identification, because this identification by others is closer to how implicit bias functions (Staats et al., 2016). Although race is not the focus of our study, Table 1 shows that the classroom consisted primarily of white students.

Table 1. Participant Demographics

	Women	Men
Black	1	0
Latinx	2	1
White	6	6

The course used extensive inquiry-based group work, which led into synthesizing plenary discussions. Students used artifacts (e.g., rulers, string, plastic panes, dynamic geometry software) to explore geometry. The course was co-taught, with the secondary instructor also collecting data as a researcher. Students participated in consistent groups in the first half of the course, and were reassigned once at the halfway point (such that two members of each new group had been in a group together before, and the remaining two members came from different groups).

The course had three main units. Physical and spatial projective geometry (unit 1) was built around the problems that gave rise to projective geometry. Synthetic projective geometry (unit 2) focused on axioms and proofs, including major results such as Desargues' Theorem and Pappus' Theorem. Analytic projective geometry (unit 3) focused on homogeneous coordinates in projective geometry.

[1] Hispanic Serving Institution is a federal designation in the United States indicating that a university has an enrollment of 25% or more Hispanic full-time equivalent students and significantly serves high-needs students, among other criteria. (see https://sites.ed.gov/hispanic-initiative/hispanic-serving-institutions-hsis/)

Data Sources

Five cameras (four on groups, one on the whole class) were used to capture talk in various classroom spaces over 27 class sessions (the first 2 sessions, as well as the 24th session, were omitted from the dataset, as the semester was getting started up and the class had guest speakers, respectively). We distinguish between talk in three such spaces: private *group talk* that took place at tables during the inquiry activities, private *side talk* that occurred at tables during plenary discussions, and *public talk* that was audible to the whole class during plenary discussions. Side talk was captured by the group cameras that continued running during the plenary.

We sampled a subset of the data for analysis: for each of the three class units, two tasks were chosen at random (six tasks total). Each task had five videos: four group videos and one whole-class video, for a total of 457 minutes of interactions. A total of 2968 sequences (described in the following section) were coded. For our quantitative analysis, we only considered "on topic" talk, because it represents the mathematical contributions made in class. There were 2027 on-topic sequences, or 68% of the talk (1731 for group talk, 186 for side talk, and 110 for public talk). Our qualitative analyses draw from all forms of talk.

Analysis

Data were coded using the equity analytics approach, which focuses on identifying quantitative patterns of equity and inequity in classroom participation (Reinholz & Shah, 2018). These quantitative data draw attention to how different groups of students—or individual students—within the same classroom may have differential access to learning opportunities. Equity analytics is designed to answer questions such as: Do women and men participate in a classroom in similar amounts? Or, how does the quality of teacher questions asked to Black students differ from questions asked to Asian students? A focus on quantitative data makes it easier to see subtle patterns that may otherwise go unnoticed. In addition, equity analytics can be used in conjunction with qualitative analyses to provide multiple perspectives on classroom interactions. We used the classroom observation tool EQUIP, which provides a specific approach for generating equity analytics (Reinholz & Shah, 2018). In EQUIP, participation is coded in terms of sequences, which constitute a chunk of student participation uninterrupted by another student in the class.

In this paper, we used a subset of the seven EQUIP codes, to focus exclusively on students, because groups primarily interacted without the instructors. Our student focus allowed us to use the same coding scheme for group talk and public plenary talk. We used two standard EQUIP dimensions (length of talk, type of talk), and added the additional dimensions of classroom space ("group talk," "side talk," and "public talk") and on-topic talk ("on topic" vs. "off topic"). We coded student statements on the "type of talk" dimension using four categories: *what*, *how*, *why*, and *other*. *What* statements were those statements where a student stated a mathematical fact or provided an answer without providing justification (e.g., "Those lines will intersect at the infinite."). *How* statements consisted of a student providing information about a procedure, such as a list of steps to arrive at a solution to a task (e.g., "First we need to show that the lines intersect, then we can use the axiom."). *Why* statements were those where a student provided explanation or justification for a mathematical idea, a procedure, or a solution to a task (e.g., "We know those lines will intersect because of Desargues' Theorem."). We cross-tabulated coded data with demographic information from the class to generate analytics. Below, we focus on high-level contributions from students, when they made statements of 21+ words (roughly two or more sentences) and mathematical explanations, coded as *why* statements (Reinholz & Shah, 2018). The cutoff of 21 words corresponds to roughly two sentences, which is considered as a benchmark for meaningful student engagement (Daro, 2019).

To describe our results, we use the *equity ratio* (see Reinholz & Shah, 2018), which is the ratio of *actual participation* to *expected participation* for a group of students along a particular dimension of classroom discourse. Actual participation is determined by classroom observation using EQUIP. Expected participation is what one would predict based on a group's demographic representation in a classroom. The equity ratio can fall into three categories: greater than one, less than one, or equal to one. To illustrate, imagine a classroom where 40% of students are women. If actual participation from women is 60% of total participation, then the equity factor would be 1.5—this indicates disproportionately *greater* participation from women relative to their demographic representation. If actual participation from women were 30%, the equity factor would be 0.75—indicating disproportionately *less* participation. If actual participation is 40%, the equity factor would be 1.0, indicating proportional participation.

Interrater Reliability

A representative sample drawn from four of six tasks chosen at random was double coded to establish interrater reliability, with a different group sampled for each task. Thus, one hour of video was double coded, with 811 sequences (approximately 27% of the dataset). To compute interrater reliability (Table 2), we used Krippendorff's alpha (Hayes & Krippendorff, 2007). All results exceed 0.8, for good reliability, the highest category that can be achieved (Carletta, 1996).

Table 2. Krippendorff's alpha for five dimensions (N=811 cases, 2 raters)

Dimension	Participant	Class Mode	On Topic	Type of Student Talk	Length of Student Talk
Alpha	1	1	0.985	0.926	0.964

Qualitative Findings

In this section, we present qualitative analyses of classroom interactions involving women. The data reveal two main findings. First, we found evidence of women displaying mathematical competence, both in small-group discussions and in side talk. This was unsurprising, as women are obviously capable of doing mathematics. However, we found little evidence that this competence made its way into the public plenary discussions. Second, we found evidence of blatantly sexist discourse and gendered microaggressions in small-group interaction.

Women's Mathematical Competence: In Group Talk and Side Talk

Women made substantial contributions in both group talk and side talk. In particular, Fiona frequently provided high-level contributions in group talk. The following episode shows Fiona discussing the central projection of a square from a particular location on a plane. She explained how projections in the given scenario would look, an idea that many students struggled with:

1. Jason: Yeah like you said, it'd, the vanishing point was gonna be underneath the table. And so that-
2. Fiona: Well no, your vanishing point's still above the table.
3. Jason: It is?
4. Fiona: Mm hm.
5. Jason: I thought you just said it was under.
6. Fiona: Your vanishing point will continue to stay at your eye level. It's just that if you extend that board down all the way to the bottom and this is a clear table that you can reach through and start drawing this figure, it's gonna be really huge. Because you're looking down at it, so it's being projected at this kind of angle. So I'm like this, and then, it's gonna go like this (indicating with her hands how the projection of a square would elongate),
7. Jason: Okay.
8. Fiona: as you look down on the table. But I can't, just stick my hand through the table and start drawing. If I could, that'd be awesome.

Here, Fiona provided a critical explanation for understanding projection in this context – specifically that the vanishing point would always stay fixed at eye-level and the object would be projected downward, underneath the tabletop. Later during plenary discussion, there was disagreement about how the square would get projected. However, Fiona did not share her explanation in the plenary. In fact, over the duration of the course Fiona only made one *why* statement during plenary, even though during group discussion she made the most *why* statements of all study participants.

Other women also displayed high levels of mathematical competence. For example, on one occasion Trisha discussed with her group the conditions in homogeneous coordinates that indicate whether a point is on a particular line.[2] In the following episode, Trisha engages with Dr. R and explains that if the dot product between the triples representing a point and a line is zero, then we can think of the point under consideration as being the same point, but represented on a different plane. In doing so, Trisha makes an important *why* statement:

1. Dr. R.: The question is, you have two vectors, A and B, what is the condition that makes the parentheses B, the point, be on the bracket A, the line A. So here you have the bracket A and the parentheses B.
2. Trisha: So if it's parallel?
3. Dr. R.: What is parallel to what?
4. Trisha: If point B- See how that point makes a new line B?
5. Dr. R.: Yes, yes, yes.
6. Trisha: Or line A, line A. So you're saying if it's parallel to the plane?

[2] In this class, homogeneous coordinate notation consisted of points represented by a triple (X,Y,Z) and lines represented by a triple [X,Y,Z]. Furthermore, both points and lines belong to equivalence classes, and as such, can be represented by more than one triple - in which case the line or point is being represented on a different plane. The criterion for a point to be on a line is if the dot product of the triple representing the point and the triple representing the line is zero.

7. Dr. R.: Parallel to the plane or perpendicular to the vector A.
8. Trisha: Or perpendicular to the vector.
9. Dr. R.: So A has to be perpendicular to B, which means that their dot product-
10. Trisha: Is zero.
11. Dr. R.: -is zero. Why don't you explain to your-
12. Trisha: It kind of makes sense because if the dot product is zero that just means, like, it's talking about- it's like the same line but just higher.
13. Dr. R.: Yes, the same- B, you mean?
14. Trisha: Yeah, it would be the same line it's just on a different plane.
15. Dr. R.: Yes.

This notion that a point can be represented on more than one plane appeared to be challenging for students to grasp, with multiple students—other than Trisha—saying in their small groups that they were confused. And yet, despite Trisha's understanding and her ability to provide an explanation, Trisha remained quiet during the plenary discussion. Instead, that discussion was dominated by one of the men in the class.

Women also displayed mathematical competence in side talk during plenary discussions. In the following episode, Trisha engages in side talk with Mike. This side talk occurred while a third student was presenting to the class about the projection of parallel lines, and suggesting that since the projection of parallel lines will converge at the vanishing point, the entire projection of a set of parallel lines would look like an X. In side talk that is inaudible to the rest of the class, Trisha explains to Mike what she believes the presenting student is trying to articulate. She uses pen and paper to illustrate her understanding by drawing two lines that start at the bottom of the paper, converge to a vanishing point, and then diverge above the vanishing point:

1. Trisha: That makes sense. So it's like, once it, goes to the vanishing point, it'll do something like, like, like, (sketching two lines converging to then diverging from a point)
2. Mike: That's what I'm thinking, yeah.
3. Trisha: Right? It makes sense.

Here, Trisha engaged in mathematical sense making by clarifying and illustrating a complex idea. Again, though, rather than contribute her explanation publicly, Trisha talks only to her group member.

Another episode of this kind involved several women in the class. During a plenary discussion about whether the projection of a parabola (from a particular location) would result in an ellipse, and whether that ellipse would touch the horizon line at a single point (a vanishing point), three women in one group—Mary, Carmen, and Isabella—engaged in side talk debating the validity of an idea that had been presented publicly. Specifically, they discussed whether the projection of the parabola would become an ellipse, and related it to the projection of parallel lines that do converge at a vanishing point. There was also a man present in this group, but he did not participate in the side talk. In this case, the women concluded that the ellipse would *not* converge to a vanishing point:

1. Mary: You know how like parallel lines intersected? Except those are straight.
2. Carmen: Yeah.

3. Isabella: This one's going out (making the shape of a parabola with her hands).
4. Carmen: Well I would just think like you're going farther, like- I don't know. I would say that they're- like you know how parallel, they get close right away, I don't think the parabola's growing fast enough (tracing the shape of a parabola with her hands). Maybe if it was like this or something (making a wider parabola shape with her hands).

In plenary discussion, the dominant perspective was that the projection *would* converge at the horizon, yet no convincing argument had been provided. These women expressed a contrasting perspective in their side talk. Their ideas might have injected provocative grist for debate, but they were never made public.

Sometimes women's side talk did enter the public space. In the following episode, the class had been attempting to explain why three points (corresponding to Desargues' Theorem in three dimensions) were collinear. To do so, students had to consider the three sets of two lines each, and explain why those lines must intersect. Here we first present the public discussion occurring between one man and the instructor, and then we present side talk between two women at a nearby table:

1. Dr. R.: Yes, but say if we extend [the lines], it turns out they will meet around here, but in terms of the axioms how do you prove-
2. Raul: Axiom one.
3. Dr. R.: -that, given this construction, they have to meet somewhere?
4. Raul: Axiom one, with two lines.
5. Dr. R.: Excuse me?
6. Raul: Two lines have to be incident at at least one point. Can you do that and do it on the edges? Or no?
7. Dr. R.: Well, look at your list of axioms.
8. Raul: (to Carla) Do you have [the list of axioms]?
9. Dr. R.: Which axiom is the one-
10. Raul: Is it axiom two? We're saying two. (to his table group) Yeah, it's axiom two. That's the one I was trying to get at.

Raul focused on choosing the correct axiom that would indicate the two lines intersect, but did not provide a mathematical explanation. During this exchange, two women at a nearby table focused in on an important aspect of the mathematical situation – specifically, that the two lines to which the instructor was referring must be coplanar lines.

1. Candace: (privately) But those two are coplanar.
2. Fiona: (privately) Yeah, they would have to be coplanar.
3. Candace: (privately) But they're coplanar. Because they're coplanar in the line of the-
4. Fiona (publicly): Oh, could we say that this part right here, from here to here, and here and here, and here to here (indicating to three lines that intersected pair-wise), makes a whole new plane? So these are on that same plane, and then two coplanar lines, by axiom two, meet?
5. Dr. R. (publicly): Yes.

During their private conversation, the two women agreed that the two lines in question were coplanar lines. This idea had not yet been introduced publicly, yet it was essential for proving that the two lines must intersect. This episode showcases the value of side talk, which may have served to solidify Fiona's understanding of the situation and given her the confidence to share the idea publicly.

Sexist Discourse and Gender Microaggressions

The episodes analyzed thus far show that women had valuable insights that rarely made their way into public discussion. While we did not interview participants on this particular issue, it may be that the classroom space did not create a comfortable atmosphere for them to publicly share their thinking. In that vein, we documented several episodes[3] during small-group work involving explicitly sexist discourse and gender microaggressions. In the first episode, students were using axioms to perform a geometric proof:

1. Isabella: Do this one.
2. Raul: Huh?

…

7. Carla: He doesn't listen to girls.
8. Raul: Huh?
9. Carla: Sexist, too.
10. Raul: Like I said before- what'd I say in the other class?
11. Isabella: What?
12. Raul: It's hard to hear her when she's in the kitchen. These aren't collinear.

Isabella began by suggesting a part of the problem to work on (line 1), and Raul responded as though he did not know what was going on (line 2). This resulted in Carla making a charge of sexism (line 7: "He doesn't listen to girls"), to which Raul repeats the same move of pretending not to hear Carla (line 8). The conversation culminates in Raul taking up and seeming to embrace Carla's charge of sexism by saying, "It's hard to hear her when she's in the kitchen."

To be clear, Raul's behavior here goes beyond microaggressions; it is blatantly sexist. Although to some Raul's comments might seem like harmless "jokes," his invocation of the "kitchen" is deeply problematic. Not only does it index the sexist narrative that a "women's place is in the kitchen," but when invoked in a mathematics class, it also implies that a women's place is *not* in mathematics. Raul displayed a form of masculinity that has a long history in mathematics education (Mendick, 2006). Indeed, Raul often dominated public discussions, and generally did not make space for his classmates to contribute during small-group work, which his peers interpreted as him not listening. Sexist interactions continued in the same group later that class session:

1. Fabrice: Dude, that pen is not even straight. At all. Yeah, like that, there you go.
2. Carla: Axiom one, "any two distinct points are incident with one line."
3. Raul: It has to be with this line. God, you're silly.
4. Isabella: I like silly better than stupid. I know that you're trying to say the same thing.

…

[3] Lines of talk that are omitted from the transcript are times at which the group is focused on the mathematical task, or instances in which the group is off topic, but in a non-gender-problematic way.

14. Fabrice: You just have a bad perspective from where you're sitting.
15. Raul: Hey, fix your perspective.
16. Carla: I will. By removing two men from the table. I think axioms one, three and seven.

In this episode, Raul dismissed Carla's contribution and called her "silly" (line 3). The word "silly" is not explicitly gendered, but how Carla takes it up suggests that to her it amounted to a gender microaggression. In the immediate aftermath of Raul's patronizing dismissal, Isabella notes that by "silly" Raul really means "stupid" (line 4). Both women in this group apparently recognize Raul's attempts to denigrate ideas. At the end of the episode, Carla engages in explicit gender talk by calling for the removal of "two men from the table" (line 16). This sheds a different light, retrospectively, on the previous interaction around the word "silly," raising the possibility that both women took up Raul's comment as a gender microaggression dismissing their mathematical competence as women. It is unlikely that he would have referred to another man in the course as "silly." Interestingly, these microaggressions and explicitly gendered talk occur almost seamlessly intertwined with the students' mathematical discourse.

As a final example, we present a third episode highlighting that sexist discourse was not only deployed by men. In the following transcript, Hakan (man) joked about Fiona's ability to be a good teacher after Fiona said that she read the homework but did not fully understand it. At that point, the conversation shifted to Fiona's current job working in a hardware store:

1. Fiona: I think that every day. I don't know what I'm going to be. I work in a hardware store and people come in and ask the stupidest questions, and I'm just like-
2. Jerry: Do you really work in a hardware store?
3. Hakan: Do you know how to stop the faucet leaking?
4. Fiona: No.
5. Hakan: Mine is dripping like crazy.
6. Fiona: I know nothing about plumbing and electrical.
7. Candace: She's just there to look pretty.
8. Fiona: Basically. Although I am a sales associate.
9. Candace: What department do you work in?
10. Fiona: All of them.
11. Candace: So pretty much you—

…

16. Candace: Pretty much you just hang out in the garden section?
17. Fiona: We don't have—well we do but it's really small. I hang out in the houseware section.
18. Candace: Where you belong.

This episode shows how women can also participate in cultures of sexism in mathematics. After Fiona reveals that she works in a hardware store, the two men in the group express surprise and also question Fiona's competence related to home repair (lines 2-6). After Fiona confirms that she "know(s) nothing about plumbing and electrical," Candace asserts that Fiona is "just there to look pretty" (line 7). The focus on Fiona's body and appearance invokes gender, particularly in the context of the discussion about hardware and home repair, which have historically masculine connotations. Fiona seems ambivalent in response to Candace's comment:

affirming it while also pointing out that she is still a “sales associate” (line 8). Candace and Fiona then end up in a conversation about which department Fiona “hangs out” in, with Candace concluding that the “houseware section” is where Fiona “belongs” (line 18).

Again, it was unclear whether Candace intended this gender microaggression as a joke, and we lack the data to discern the impact the episode had on Fiona. It is possible she brushed it off as harmless, but it is also possible that she viewed it as demeaning and offensive. Certainly, the “houseware section” (as opposed to “plumbing and electrical”) echoes Raul’s invocation of the “kitchen,” signifying a sexist narrative about where women are thought to belong and not belong.

Summary

The qualitative analysis revealed positive and disturbing findings related to the experiences of women in the class. On the positive side, there was considerable evidence of women’s mathematical competence. This indicates that in small-group spaces women were comfortable demonstrating their mathematical ability. However, in nearly all of those episodes, women’s mathematical ideas were not elevated to the public arena of plenary discussions. Further, we found evidence of sexist discourse and gender microaggressions deployed against women in small-group work. We do not have the data to claim that sexist interactions were widespread or systematic in the class. However, our finding is consistent with prior documentation of sexism across STEM education and in society more broadly that was discussed earlier. At least, we can say that sexist discourse was “in the air” and some students were actively engaged in it.

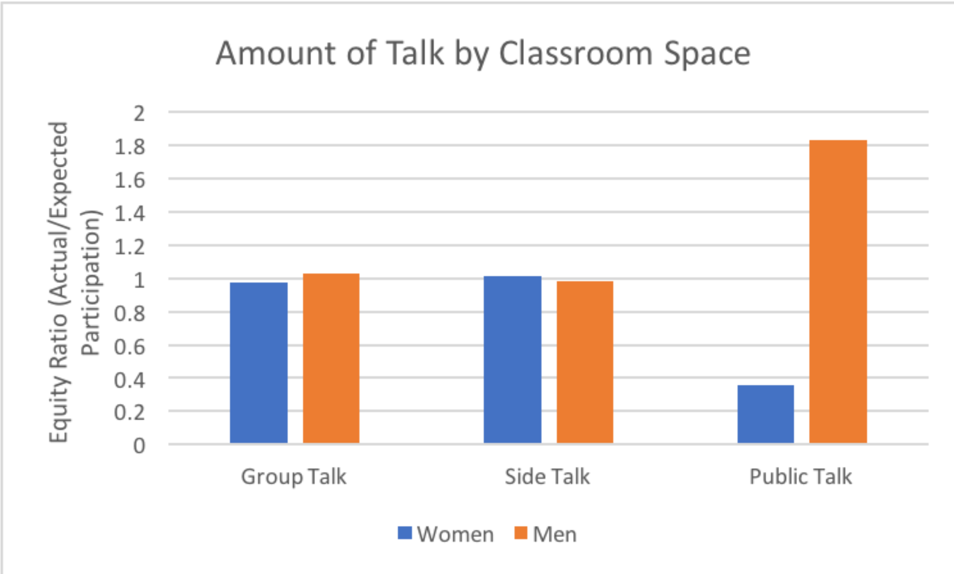
In the next section, we use quantitative analysis to discern more systematic gender patterns in the quantity and quality of participation by women and men in the class. Specifically, we extend the qualitative analysis by quantifying aspects of students’ participation in more private spaces (i.e., small group and side talk) and comparing it to their participation in public spaces (i.e., plenary discussions).

Quantitative Findings

Quantity of Participation

We first investigated the amount of participation by classroom space (see Figure 1). The differences in participation levels for public talk were pronounced. The results were significant, $\chi^2(1, N = 2027) = 60.21$, $p = 8.43 * 10^{-14}$, Cramer’s V = 0.17. Following up on the overall significance of the chi-square, we conducted post hoc analyses of the standardized adjusted residuals to investigate the significance of results for particular classroom spaces, and not just participation as a whole. For side talk, the standardized adjust residuals were 3.02 (women) and 3.22 (men), both which cross the threshold of +/- 2 for significance (Sharpe, 2015). Although these differences are statistically significant, the differences in amount of talk are small, indicating that participation was relatively proportional for men and women in group talk. The results were also significant for public talk (residuals: 8.48 women, 11.03 men), but not private talk (residuals: 0.29 women, 0.38 men).

Figure 1. Participation by classroom space.

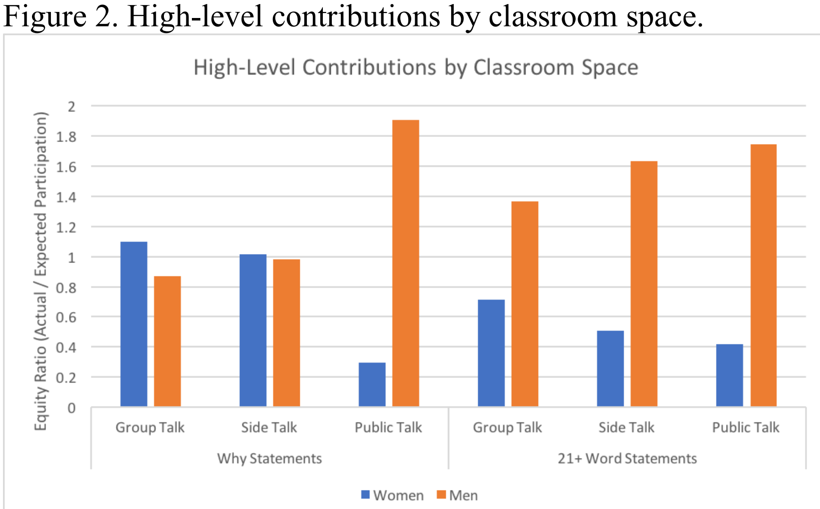


These quantitative results corroborate our qualitative analysis. Men and women participated at relatively similar levels in private talk, but men dominated the public space in plenary discussions.

Quality of Participation

To investigate the quality of participation, we analyzed high-level contributions (why statements and contributions of 21+ words; see Figure 2). The results were marginally significant for why statements, $\chi^2(2, N = 160) = 5.73$, $p = 0.057$. It is possible that the results would have been significant for why statements with a larger sample size. The results were significant for length of talk, $\chi^2(2, N = 211) = 29.37$, $p = 4.17 * 10^{-7}$, Cramer's V = 0.14. The results show that men spoke in extended contributions more often than women, across all spaces.

Figure 2. High-level contributions by classroom space.



The Instructor's Distribution of Participation Opportunities

To understand the lack of public participation of women, we analyzed how the primary instructor solicited participation during plenary discussions. The secondary instructor did not help facilitate these plenary discussions. We identified a total of 17 solicitations, of three types: (1) asking specific students to share their reasoning, (2) asking groups to share their reasoning, and (3) posing general questions to the whole class (see Table 3). We only include solicitations for new ideas, not when the instructor asked follow-up questions to a student who was already speaking.

Table 3.

Specific Student	Groups	Whole class
3	3	11

In the "specific student" category, the instructor only called on men. In one example, the instructor began by asking a group for its input, but when nobody responded, he called on a man in the group by name. In the "groups" category, the instructor said things like "we'll have this group sharing first" without specifying a particular group member. The "whole class" category consisted of general solicitations, like "other ideas?" and "how do we prove…?" After each solicitation, the instructor called on the first student to raise their hand, rather than waiting for more students to raise their hands. In all instances except for one, the first student to raise their hand was a man. Given this lack of wait time, men dominated the plenary discussions.

Discussion

Our findings highlight the complexity of gender equity and inequity in inquiry-based classrooms. There was strong evidence of women's mathematical competence in both the qualitative and quantitative data. We found that women frequently offered complex mathematical explanations and participated in rich mathematical discourse, particularly during small-group work and side talk. In fact, women's participation in mathematical discourse was usually proportional and sometimes disproportionately greater relative to their representation in the class. However, women's mathematical participation in these more private spaces rarely was elevated to the public arena, which was dominated by men. As a result, women's mathematical competence remained largely hidden.

The evidence in support of gender equity in small-group work was a positive sign. However, qualitative analysis revealed numerous instances of both flagrant sexism and more subtle gender microaggressions. Although there were insufficient data to claim that sexism was systematic across all groups, this finding is consistent with prior research on gender in mathematics education (Langer-Osuna, 2011; Mendick, 2006) and on sexist culture more generally (Sue, 2010). This finding troubles the idea that small-group work automatically produces equity. To the contrary, women in this class demonstrated mathematical competence in spite of sexist interactions. In fact, research has shown that alongside its benefits, small-group work can provide fertile ground for inequities to emerge (Engle, Langer-Osuna, & Royston, 2014; Langer-Osuna, 2011, 2016; Shah & Lewis, in press).

One limitation of the study was the relatively small sample size of 16 students in one mathematics classroom. This small sample size, and the relatively homogeneous racial demographics of the class, prevented us from conducting intersectional analyses of race and gender. Moreover, we recognize that a single classroom cannot necessarily speak to the experiences of women in all mathematics classrooms. Nonetheless, our analyses provide an important example of marginalization at the level of classroom interaction in post-secondary mathematics, and given that this course was taught by a highly-experienced instructor, we surmise that these results are not uncommon. This is an area for future research.

Despite these limitations, this study raises an important question: Why was there such a large disparity in women's mathematical participation between public and private spaces? Our analysis showed that the instructor made available fewer opportunities for women to participate in public discussions (e.g., by rarely calling on women). Whereas in small groups women can more easily claim the conversational floor, in plenary discussions the instructor has far greater authority to distribute participation opportunities. There was no evidence that the instructor in this class was overtly sexist. However, no one is immune to implicit bias (Staats et al., 2016). In the absence of explicit attention to issues of gender equity, an instructional setting will likely perpetuate the sexist status quo.

Second, it is possible that shyer personalities or insufficient confidence dissuaded the women in this study to participate publicly. However, we find this less plausible for two reasons. First, we observed that women in private spaces were quite assertive in making mathematical contributions. Second, "shyness" and "confidence" can be thought of as situated and socially constructed. That is, rather than a lack of confidence, perhaps women lacked confidence that the instructor or the classroom's norms would protect them against sexist discourse in the public space. Gender microaggressions or outright sexism in a semi-private space is one thing, but imagine if a woman had been told she belonged in the kitchen in front of the entire class—it would be reasonable for students from marginalized groups to worry about taking a risk by participating in public ways.

Implications

This study has several implications for mathematics instruction. First, instructors should think critically about how they use small-group work. The data here suggest that instructors need to actively ensure that these spaces become springboards for public participation, so that women's mathematical competence is widely and publicly recognized. This is particularly important as recognition by others may play an impactful role in women's mathematical interest and persistence (Piatek-Jimenez, 2015). Second, we should not assume that small-group work is automatically equitable for women. While there is evidence that women-majority groups like those in this study can contribute to greater equity (Dasgupta, Scircle, & Hunsinger, 2015), we still documented many instances of overt sexism in group interactions.

Third, small-group work is not the only type of private space in classrooms where mathematical discourse happens. Instructors should also consider how to legitimize and structure opportunities for mathematical side talk. Although side talk is often seen as distracting or disruptive, this study shows that it can also have value for learning. Finally, and perhaps most importantly, instructors must confront and account for sexism in mathematics education. Professional development should be made available to instructors and students regarding the overt and subtle ways women experience sexism in mathematics classrooms.

In future research, it is imperative that we continue to document women's mathematical competence as a counternarrative to sexist discourses that undermine it. Along those lines, interviews with women in mathematics about their experiences would provide an important perspective complementing observational studies. In these ways, research can play a role in shifting discourses about women in mathematics by making visible the complexity of their experiences.

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