Using analytics to support instructor reflection on student participation in a discourse-focused undergraduate mathematics classroom

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This paper describes a method for using instructional analytics to support changes in teaching practice. The study took place in a discourse-focused undergraduate mathematics classroom taught by an experienced instructor. Using the classroom observation tool EQUIP and social network surveys, we generated quantitative data on patterns of student participation in this classroom that were presented to the instructor at the end of the semester. Our analysis focuses on how the instructor made sense of these analytics and ultimately changed his teaching practices in a future semester. This paper provides insight into how three data sources can be triangulated—classroom participation, student experiences, and teacher perspective—to better understand discourse-based participation in undergraduate mathematics. We also discuss the implications of using this methodology for faculty professional development.

Keywords: Instructor/lecturer professional development; Classroom interaction and discourse; Postsecondary; Mixed Methods (Quantitative and Qualitative)

Introduction

There is increasing momentum for active learning courses in mathematics. The ways in which such courses support rich student engagement are well-documented across a variety of studies (e.g., Larsen, Johnson, & Bartlo, 2013; Rasmussen & Kwon, 2007; Reinholz, 2015; Wawro, Rasmussen, Zandieh, Sweeney, & Larson, 2012). As a result of this engagement, research shows that active learning courses benefit *all* students in terms of course grades (Freeman et al., 2014). In addition, the impact of such courses extends beyond the courses using active learning to future courses, though the mechanisms which provide these benefits are not well-understood (Kogan & Laursen, 2014).

Although active learning courses are largely understood to be beneficial to students, more work is required to understand how participation plays out in these classrooms in everyday classroom interactions. For instance, what types of questions might an instructor ask of their students and how do students take up those questions? Do students and their instructor perceive participation in these classrooms in a similar way? And if so, do these perceptions reflect actual participation in the class? If not, is there a way to provide useful analytics to an instructor about what is going on in their classroom so that they can adjust their practice? Teaching in an active classroom a highly complex activity, and simultaneously monitoring the types of amount of participation of all students while in the act of teaching is challenging. Thus, it would be valuable if we could provide useful information to an instructor about their own practice.

In this article, we explore this possibility by investigating patterns of participation in an undergraduate mathematics classroom taught by an experienced instructor that made extensive use of active learning. Here we triangulate between data from three sources: classroom participation patterns as measured using the observation protocol EQUIP (Reinholz & Shah, 2018); students' nominations of important contributors; and interviews with the instructor about his pedagogical intentions and perspective on the observational data. This is the first major contribution of this paper: closely examining how a triangulation of these three data sources can provide greater insight into classroom participation.

The second contribution of this paper is an existence proof of using educational analytics to support an instructor's reflections, which resulted in concrete changes to the instructor's teaching practices. Whereas prior studies have used observational data to make claims about classroom participation patterns, we are not aware of studies that have focused on how instructors make sense of those data and how instructor sensemaking might influence teaching practice in undergraduate mathematics. Additionally, we include and analyze data on students' perceptions of classroom participation. By leveraging data from classroom observations, student experiences, *and* teacher perspectives, we produce a more comprehensive picture of participation in an active classroom, which we see as a key step in continuing to improve teaching practice. This paper builds on innovative methodologies for studying classroom participation and supporting instructor reflection. We conclude by making recommendations for future research in this vein and how it may lead to future professional development efforts involving instructor reflection on student participation data. The contributions of this paper can be formulated in terms of two research questions that we investigate:

1. What are the affordances of triangulating three data sources—student experiences, classroom participation, and teacher perspective—to better understand participation in a discourse-focused undergraduate mathematics classroom?
2. How can analytics that describe patterns of student participation be used as a tool to support instructor learning, and ultimately result in changes to teaching practices?

Conceptual Framework

Research shows that talk and learning are related (Hufferd-Ackles, Fuson, & Sherin, 2004; Michaels, O'Connor, Hall, & Resnick, 2010). However, not all talk is equally beneficial for learning. For instance, when students only make low-level contributions, they miss out on richer opportunities to learn (Mehan, 1979). Research also highlights the ways in which instructors can help support high-level discussions (Hufferd-Ackles et al., 2004; Michaels et al., 2010; Rasmussen & Kwon, 2007). Still, some students have more opportunities than others to participate in these discussions (McAfee, 2014; Sadker, Sadker, & Zittleman, 2009). To conceptualize the complexity of classroom participation, we utilize and triangulate between three interrelated perspectives: *classroom* participation, *student* experiences, and *teacher* perspective. Each perspective provides unique and important insights (see Figure 1). When used in conjunction, we argue that they provide a more robust understanding classroom participation.

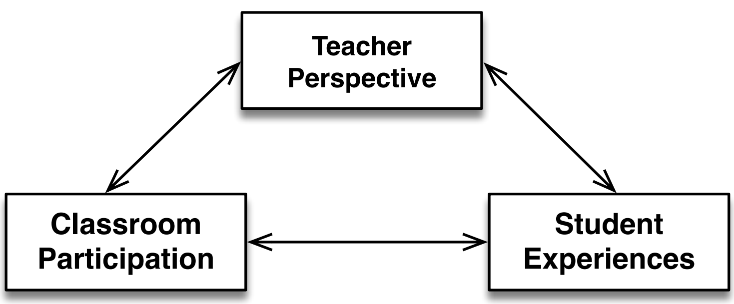
With respect to classroom participation, it is important to track patterns in which students are participating, how they are participating, and how participation opportunities are distributed amongst students. Here we analyze classroom participation patterns using a classroom observation tool called EQUIP (Reinholz & Shah, 2018). EQUIP tracks quantitative patterns in student participation and the opportunities teachers make available for students to participate in classroom discourse. Tracking these patterns is important because they can reveal differential opportunities to learn through participation in classroom discourse. In the present study, we use observational methods to systematically track classroom participation patterns in whole class discussions.

With respect to student experiences, we recognize that observational data can only tell a partial story. Students have their own subjective experiences in the classroom, and these experiences must be taken seriously (Lubienski, 2000; McGee & Martin, 2011; Stinson, 2008).

For instance, even if certain students receive a proportionate share of participation opportunities, it would still be a problem if they perceived that they did not have sufficient opportunities to participate, or if they experienced the classroom environment as hostile. Whole-class discussions are public fora where students have the opportunity to be identified as competent by the rest of their peers (Cohen & Lotan, 1997), so it is important that students themselves feel that they have opportunities to participate in such settings.

With respect to teacher perspectives, it is important to understand how teachers make sense of participation-related patterns in their classrooms. Teachers have deep professional knowledge of both their own practice and their own students. Eliciting their perspectives can provide additional insights into patterns identified in observational data. Further, giving teachers opportunities to reflect on participation-related patterns in their classrooms has the potential to effect changes in their practice. In fact, this was a key question driving the present study. Teachers have certain pedagogical intentions reflected in how they structure their practice, which may or may not be realized in actual classroom participation patterns and in how students experience the classroom. In that sense, we view these three perspectives as interrelated.

Figure 1. Three perspectives on participation.



Methods

Context and Participants

The study took place in an upper-division undergraduate mathematics course aimed at future teachers. There were 8 students in the course, and most of them had previously taken mathematics courses together. Students were assigned to three groups which they retained for the entirety of the course: 1) Delores, Luther, and Bradley; 2) Ron and Casey; 3) Gregg, Kathryn, and Vicky (cf. Table 1). This course consisted primarily of students working on challenging problems and then discussing their work as a whole class. The gender presentation of the students was determined by the instructor based on their extensive interactions.

Table 1. Group membership.

	Women	Men	Total
Group 1	1	2	3
Group 2	1	1	2
Group 3	2	1	3
Total	**4**	**4**	**8**

Tools and Data Sources

There were three main data sources for this study, one of each associated with each of the three perspectives*: classroom participation, teacher perspective,* and *student experiences*. To capture *classroom participation*, a single video camera was used to record lessons. Because these data were initially collected for another study, there was only a single camera focused on one of the groups; accordingly, we limited our analyses to whole class discussions. We had video from 22 of 30 class sessions (75 minutes each) for analysis. We could not analyze the other class sessions either due to missing data or because a substitute instructor was teaching the class. From this corpus of video, we took a sample across all of the lessons, for approximately 11 hours of video total, or about 1/3 of the total video available for coding. This was not a true random sample, but it was still sufficient for the purposes of providing analytics to an instructor for their reflection on, which is our focus. These videos allowed us to code discourse during approximately 8 hours of whole class discussions to see who contributed to the whole class discussions as well as to understand the nature of their participation.

To access the teacher perspective, we conducted two post-hoc interviews with the course instructor. The goal of the first interview was to understand the instructor's beliefs about teaching, to investigate his perceptions of the classroom discourse, and to gain insight into how he made sense of the data related to participation patterns in the class (i.e. the analyses described above). We draw on four areas of focus from the interview: (1) teaching philosophy and overall approach to learning; (2) predictions about participation patterns based on teaching that semester; (3) interpretations of the analytics; and (4) reflections on teaching for the future. During part (3) of the interview we showed actual data from the classroom for the teacher to reflect upon; these same data are included in this article. A year after the first interview, we had a quick follow-up interview with the instructor, in which he described changes in his teaching practices.

The final data source focused on *student experiences*. To understand student experiences, sociometric surveys were administered during each class period. These surveys asked two sets of questions: (1) was there a student in your group who said something today **during small group work** that significantly influenced your thinking? Who was it and what did they say? Please give as much detail as possible; and (2) was there a student outside your group who said something today **during a whole class discussion** that significantly influenced your thinking? Who was it and what did they say? Please give as much detail as possible. These two questions provide insight into who students perceived as major contributors, to triangulate with actual data on participation. Students' descriptions of what was said and how it was significant to them were not consistently detailed enough to compare their impressions of meaningful contributions to the researcher's interpretations, but they consistently identified *who* made the contributions they valued. Only students that were mentioned by name were counted (i.e. "Gregg's group" counted for Gregg only, and "Gregg, Vicky, and Kathryn's group" counted for all three of the students in Gregg's group). As mentioned above, these surveys were initially collected for another study, so one limitation our of present work is that we did not have more detailed probes of student experiences.

Analytic Procedures

The classroom videos were the classroom observation tool EQUIP (Reinholz & Shah, 2018). Coding involves breaking whole class discussions into a unit of analysis called a sequence, which we define as a string of talk by a given student—often with a teacher—that is uninterrupted by another student (i.e., a new sequence begins each time a new student participates). Thus, a sequence may consist of multiple turns by the same student, with back-and-forth between the student and the teacher. Each sequence is then coded along seven dimensions: discourse type, length of student talk, type of student talk, teacher solicitation method, wait time, quality of teacher solicitation, and teacher evaluation of student ideas. Once sequences are coded, EQUIP cross-tabulates consequential dimensions of classroom discourse with demographic information to identify how the amount and nature of student participation are distributed. This allows analyses to draw attention to participation at three levels: the whole class, groups of students, and individual students. However, given the small sample size in the present study, we refrain from using group-level analytics, and instead focus only on whole class and individual students. This is a limitation of this study that we are addressing in ongoing work.

A single experienced coder was responsible for coding this dataset. This coder is an expert with EQUIP, who was part of the five-member team on a prior study that achieved 80% interrater agreement on a reasonably large dataset (Reinholz & Shah, 2018). As such, we consider this coder a reliable user of EQUIP for this work. A speaker could be identified in 358 of 392 identified sequences (91%). The other contributions were discarded, as the speaker could not be identified, so the participation could not be linked to a particular student.

Teacher interviews were analyzed to look for emergent themes. Rather than attempting to code the entire corpus of data, we draw from the interviews to provide salient quotes that relate to the classroom analytics provided. Members of the research team independently read the interviews and discussed themes with one another before coming to a consensus interpretation. We also conducted member checking with the instructor for accuracy of our written narrative.

Student surveys were analyzed using Social Network Analysis (SNA; Grunspan, Wiggins, & Goodreau, 2014). SNA refers to the study of social relationships between actors in a given setting and is often carried out using techniques from graph theory. In our analyses, each student is treated as the node of a graph, and each nomination from the survey (i.e., Kathryn reporting that Luther said something important) is treated as an edge. Survey data from the 22 coded class periods was combined so as to have a single network, so that edges were weighted to reflect the total number of nominations from each student to each other over the entirety of the course. To identify the relative importance of students (in each other's eyes) we use the total number of nominations each student received and report how many nominations each student provided.

Findings

The results are divided into three sections. First, we provide a picture of the classroom where participation is aggregated across all students, to get a sense of the general nature and quality of mathematical discourse in this class. Next, we look at how different individual students actually participated in such discourse. Finally, we provide a brief description of the instructor's reflections on their changed teaching practices a year later. Excerpts from the instructor interview are interwoven throughout the findings to show more clearly how the instructor explained or made sense of the analytics.

Instructor Goals and Aggregate Analytics

The instructor described a classroom driven by discourse as a tool for learning mathematics. He described his goals to create a classroom space driven by student thinking,

> I think hard about the social aspect of the classroom and particular social norms [where] students are routinely explaining their thinking, students are listening to and trying to make sense of other students' thinking, trying to build on students' ideas and extend the mathematics from students' ideas.

The instructor's vision of a student-centered classroom contrasts a more traditional teacher-centric classroom focused on delivering information to the students. Instead, the instructor aimed to get student ideas out on the table and use them as a tool for building mathematical understanding. The instructor elaborated with specific discourse moves he used to support this goal. One technique he used was,

> just being interested and curious of their thinking, [asking] so how are you thinking about this, and asking them to sort of explain their reasoning, however tentative. So I try to just have questions which are not always just closed ended, looking for an answer but actually trying to understand their thinking, and also make it a safer space for them to share their thinking even if they're not sure about it. And another goal of having them listen to and try to make sense of other students' thinking, I often try to use this teacher move, discourse move, of "Dan, tell me what you think Denise is saying here," "can you say in your own words what Jose just said?" And that is not a natural discourse move, no one sits at the kitchen table and says "Mom, can you say in your own words what Dad just said?" So it's something one has to practice and put into their repertoire, if you will.

As the instructor describes, he aimed to create a "safe space" for students by giving open-ended questions and withholding his own evaluations as an instructor. Moreover, he used discourse moves to support student-student crosstalk, which is considered an important characteristic of a productive discourse community (Michaels et al., 2010). We note that this type of talk is not captured by the coding process in EQUIP. The instructor further elaborated the use of physical body moves,

> I'll have students come up to the front and present and share their work and so then I try to step to the side and intentionally not look at them, because if I look at them they're going to start looking to me and talk to me, and I don't want that, I want them to talk to the class. So physically what you can do with your body and your gaze can foster students paying attention to each other and having cross student talk rather than always to the teacher, back to the teacher.

The actual classroom-level data were consistent with the teacher's description of his approach. Looking at student talk (see Figure 2), more than 25% of student contributions were explanations (why statements). This means that during whole class discussions, students were not just simply giving answers (what statements) or saying what they did (how statements), but actually justifying their mathematical reasoning. This is a desirable situation, as giving content-related explanations helps improve learning (e.g., Chi, De Leeuw, Chiu, & LaVancher, 1994; Henningsen & Stein, 1997; Lombrozo, 2006).

Moreover, most student contributions were around one sentence in length (5-20 words) or longer (21+ words). This highlights a classroom in which students generally explained their thinking using extended contributions. This contrasts an Initiate-Response-Evaluate (IRE)-based classroom where students provide short answers consisting of only a few words (Mehan, 1979). The primary solicitation method was calling on students ("Called on"), as opposed to students participating without being called on ("Not called on"). The teacher often intentionally chose which students to ask questions to, asking a question and calling on a student at the same time.

Figure 2. Dimensions of student talk for individual students.

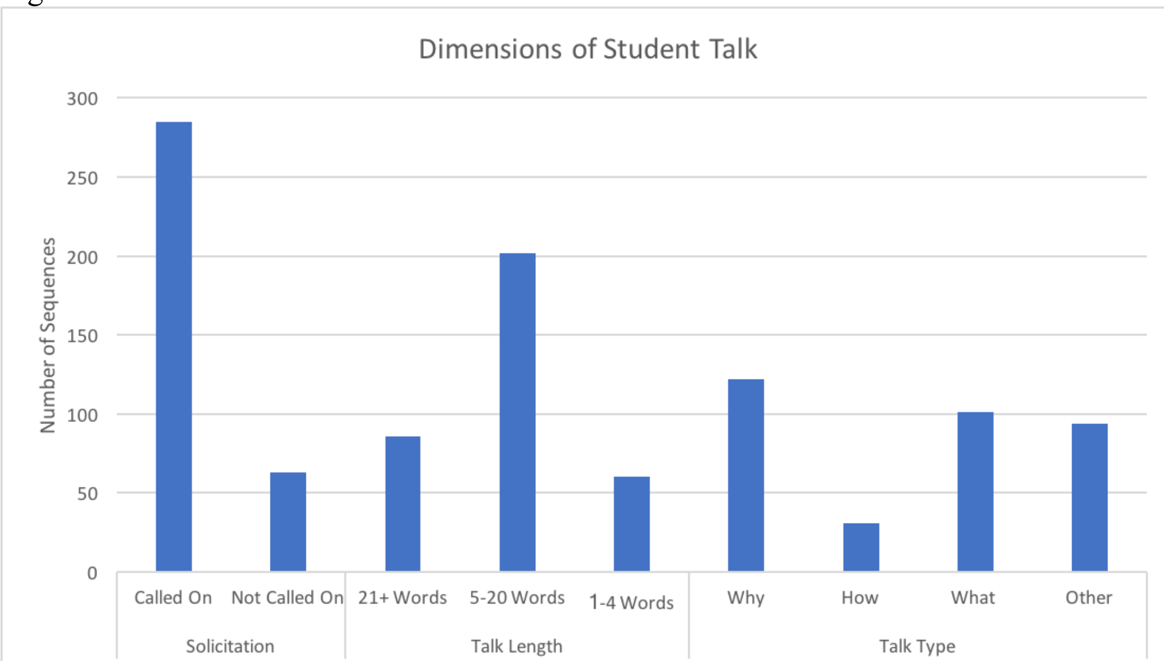


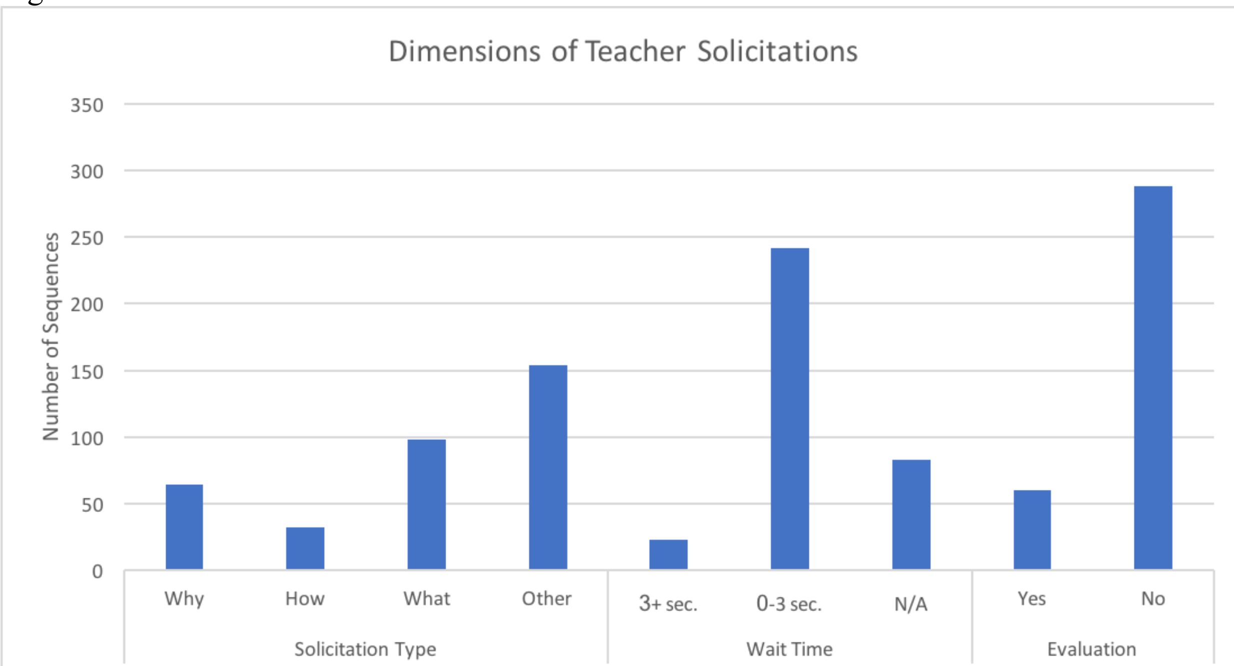
Figure 3 shows teacher solicitation characteristics, which describe the ways in which the instructor prompted students to participate. Note that many instructor solicitations are categorized as "other." Many of the items coded as such were open-ended questions (e.g., "what do you think about what Delores said?") which serve to promote student-student talk, one of the instructor's stated goals. The prevalence of these open-ended questions in this particular instructor's discourse style is the reason we found many more "other" questions, as compared to other classrooms we have studied (e.g., Reinholz & Shah, 2018). Moreover, we see that the instructor rarely evaluated student ideas. Instead, he created space for students to evaluate and respond to one another's ideas.

In EQUIP, wait time is operationalized as the time between posing a question and choosing a student to answer that question, also known as Wait Time Type 1 (Ingram & Elliott, 2014; Rowe, 1986), which research shows generally improves the quality of discussions and broadens student participation. We found that this instructor used little wait time for most of his questions, because his discourse style involved calling on students at the same time as he posed his questions. Upon reflection, the instructor found the level of wait time surprising,

> I also was struck that my wait time wasn't that long. I thought I was a little bit better with wait time but oh gosh, I was always a few seconds or something or whatever it was.

Here we notice an instance where the analytics were different from what the teacher expected, which provides an opportunity for reflection on teaching practices. The instructor's strategy of choosing who to ask questions allowed him to purposefully select students to share, but also resulted in a diminished wait time. The instructor's expressed surprise suggests that he may, in future, adjust his questioning patterns to also incorporate more wait time.

Figure 3. Dimensions of teacher solicitations for individual students.



As described above, the instructor emphasized the development of norms in the classroom in which students knew that they needed to explain their thinking. Table 2 and Figure 4 show how the type of student response related to the type of teacher solicitation. Here we see that regardless of question type, students responded with why answers with considerable frequency, corroborating the instructor's account. Additionally, students did not respond with what answers when asked why or how questions. As the instructor noted,

> Interesting that when I asked a how question I still got some whys. (laughs) Well it's kind of cool that there's so many whys spread out, all types of questions, right, which does speak to the norms of the classroom.

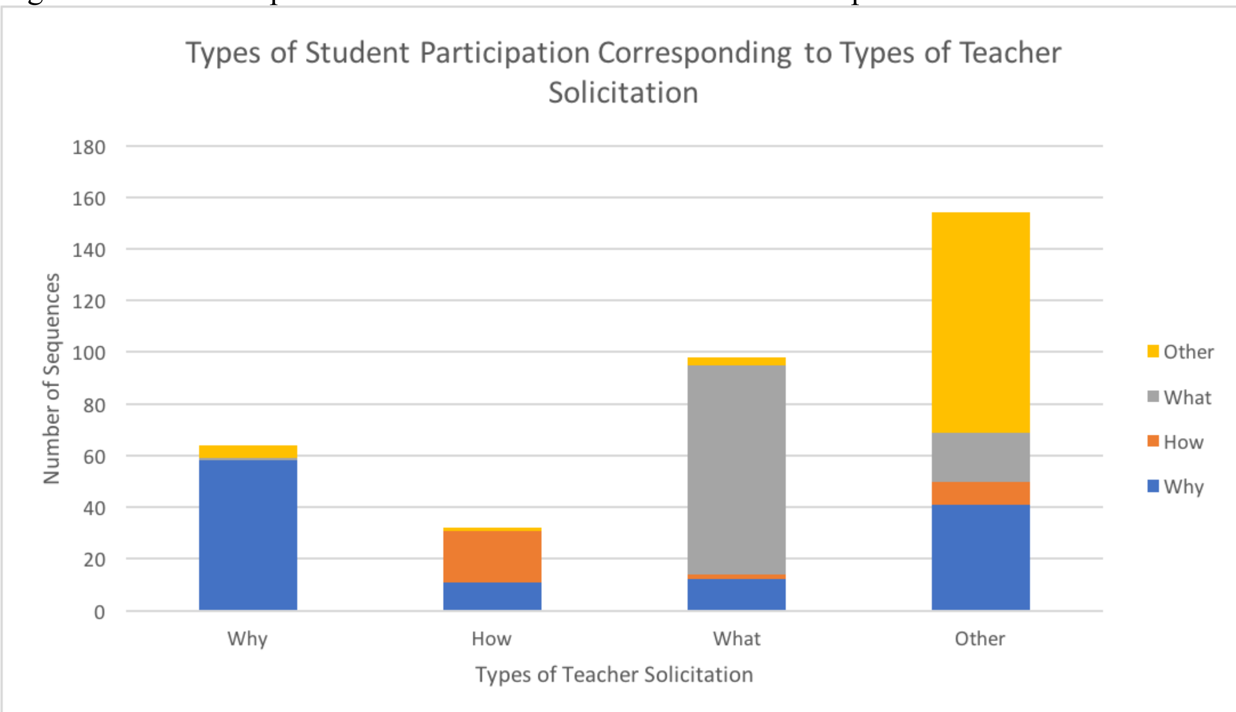
This is an instance that shows the instructor had well-established norms for participation, which agreed with his stated goals for the course and his perception of what was going on. Regardless of the types of questions asked, students seemed to know that they were expected to explain their reasoning – and were able to do so.

Table 2. Relationship between teacher solicitation and student response

Student Response	Teacher Question				
	Why	How	What	Other	Total
Why	58	11	12	41	122

How	0	20	2	9	31
What	1	0	81	19	101
Other	5	1	3	85	94
Total	64	32	98	154	348

Figure 4. Relationship between teacher solicitation and student response.



The above results paint the picture of a classroom driven by student discourse. We see consistency in the instructor's attention to student thinking and the types of student participation that were present in the class. It is evident that the specific discourse moves and other techniques that the instructor leveraged were aligned with student-driven mathematical discourse in which students regularly explained their thinking. This is no simple feat, and even seasoned instructors often struggle to adopt student-centered teaching strategies (e.g., Speer & Wagner, 2009).

Disaggregated Analytics by Individual Students

Beyond a holistic picture of the classroom, we were interested in how participation opportunities were distributed amongst different students. When asked about how he promoted the participation of individual students, the instructor responded:

> I think about it like in a single class period not every student all the time is going to have a chance to contribute, so I have to be conscious of okay, this week who didn't get a chance to talk, and then gosh, let me make sure that I open up some space for those people or person to contribute in other class periods.

As the above quote indicates the instructor made use of purposeful calling to try to include all students in the space of whole-class discussions. This was evident from the aggregate analyses, in which the instructor generally chose which student he wanted to answer particular questions. In addition, the instructor has some specifics techniques he used to help promote equitable participation. He described this as follows,

> You know, there are times where students have come to me and are like I'm just really shy and I don't want to talk, so I try to say things like I know I'm really bad at Spanish and the only way I get good at it is to practice it, and talking in class and expressing your thinking is like trying to learn a language, so I appreciate it, because I always feel terrible trying to speak Spanish because I'm crap at it so I get where you're coming from. I know that it's uncomfortable but this is necessary for growth, so help me figure out what works for you. That never really goes very well but I try, you know. (laughter)

This quote highlights the instructor trying to connect with the students at a personal level and empathize with their experiences. Figure 5 shows that all students did indeed participate in whole-class discussions. In fact, even Delores, the student who participated least, had a total of 23 sequences. Recall that we analyzed 22 class sessions, and only a portion of each class session was analyzed. This suggests that most students would have contributed at least once in a whole-class discussion during any given class session. Still, we note that some students (e.g., Gregg and Luther) did contribute more than others. As described above, while the instructor was mindful of which students participated, he did not have a systematic way to keep track of student participation. Thus, these differences in amount of participation may in part be due to the difficulty of tracking student participation simply by remembering who participated, rather than having a more systematic way to track participation (e.g., using EQUIP analytics) or a method to ensure distributed participation (e.g., random calling).

This paper is restricted to analysis of vocal participation in whole-class discussions so we cannot capture all ways of participating. For example, students in this course worked in small groups throughout the course. This provided more opportunities for each student to contribute to the mathematics developed by the class, even if they did not bring the mathematics to the rest of the class themselves. It is also worth noting that participation alone does not capture more nuanced aspects of participation. For instance, Delores, the student who had the lowest level of participation, actually had a graph named after her and attributed to her as the “Delores graph.” Students frequently referred to this graph by her name, which is indicative of assigning authority and status to her as a contributor to the class (cf. Cohen & Lotan, 1997; Engle & Conant, 2002).

The instructor was also aware that some students participated less than others as shown in Figure 5, and he made efforts to help include them more into the discussions, as described above. One way that the instructor worked to support more students to participate was by promoting student-student engagement. However, he described some of the challenges associated with this as follows,

> [there was] one student in particular [who] had a real disposition towards not listening to other students, and I knew this because at the end of each class I would give them a short reflection and there would be two parts to it: who in your group influenced your thinking and what did they say, and then who in the whole class discussion influenced your thinking and what did they say, and this particular student would almost never say anyone in the class said anything interesting. He would always just say "well the professor said this and that was really good." So that was always a challenge for me, to

work with him to see if he could develop a sense of appreciating someone else's ideas other than mine.

This quote highlights how teacher perspectives may not always be visible in certain aspects of the data, given the complex relationships between individual students, the classroom space, and other external factors.

Figure 5. Participation by individual students.

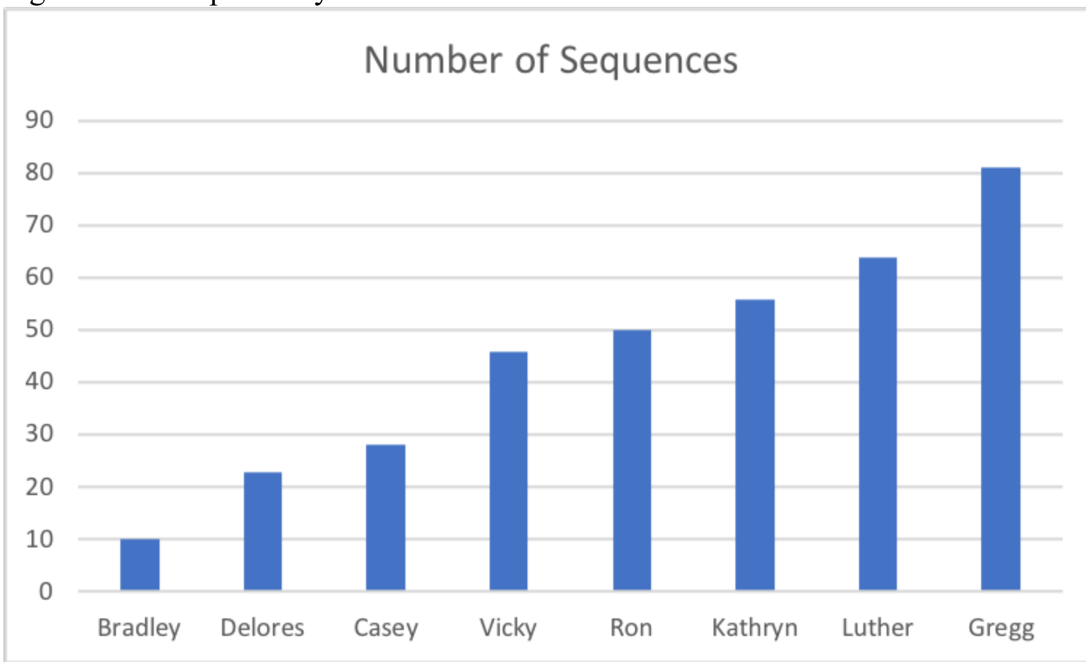


Table 3. Student nominations for valuable contributors.

	Whole Class		Small Group	
Nominations	**Received**	**Gave**	**Received**	**Given**
Gregg	60	23	36	26
Kathryn	21	22	21	21
Vicky	22	28	17	27
Luther	58	27	32	10
Delores	5	23	5	20
Bradley	1	18	7	14
Ron	14	22	12	16
Casey	8	26	16	12

To better understand participation at the individual level, we used social network analyses to see how individual students perceived the participation of their peers. Students were asked to identify one person within their group and one person outside their group each day, but sometimes they named no one, and sometimes they named multiple people. Table 3 provides a list of how often students nominated their peers as valuable contributors and how many nominations they received. Each student was nominated at least once by people outside their group, further reinforcing the notion that this was a participatory class where students listened to each other and tried to make sense of each other’s thinking. It is also clear that Gregg and Luther received the most nominations from the whole class (60 and 58, respectively) by a wide margin, almost a factor of three. We further note that, when students nominated entire groups, it was usually "Gregg's group" or "Luther's group" as opposed to naming all group members. In the small groups their margin was smaller, but these two still emerge as leaders. Thus, Gregg and Luther were seen as the most influential on their peers' thinking, consistent with their having the most contributions to the class discussions (see Figure 5).

Sociograms of the nomination networks from class are shown in Figure 6 (whole class) and Figure 7 (within group). In these figures, the thickness of the edges have been weighted proportionally to the number of nominations between a certain pair, with directionality preserved, and the vertices are sized proportional to the total number of nominations each student received. From these we can see not only who received the most nominations, but where those nominations came from. Of particular interest, of course, is to identify from whence Gregg and Luther's dominance comes. Recall that students could not select members in their group for the "out of group" question, so they had 5-6 people to choose from on any given day. Gregg, Kathryn, and Vicky overwhelmingly nominated Luther, giving him 73% of their total nominations. Luther, Delores, and Bradley spread their nominations out a bit more, but gave 47% of their nominations to Gregg and 21% to each of Kathryn and Vicky, his groupmates. Ron and Casey were the group who could select either of the prominent figures, and interestingly they gave 58% of their nominations to Gregg and only 10% to Luther. We do not have enough evidence to suggest an explanation for this phenomenon, but it does indicate the complexity and nuance of participation patterns in inquiry-oriented courses. Not only are there differences among who participates and how, there are differences in who notices and values that participation. By and large, however, this data corroborates the findings from the EQUIP analyses and the instructor's understanding of the class. We also note that the small group participation was not analyzed with EQUIP given our data limitations, even though this was a large component of participation in the course.

Figure 6. Whole class network graph (nominations limited to members of other groups).

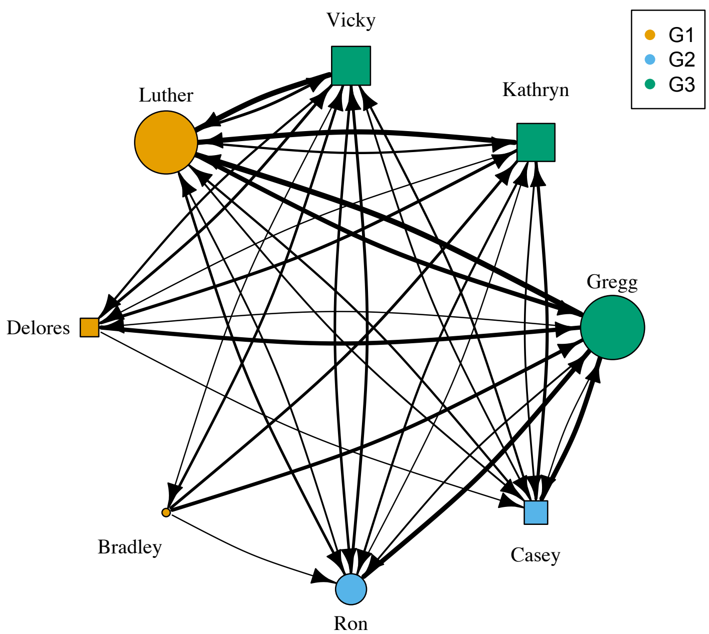
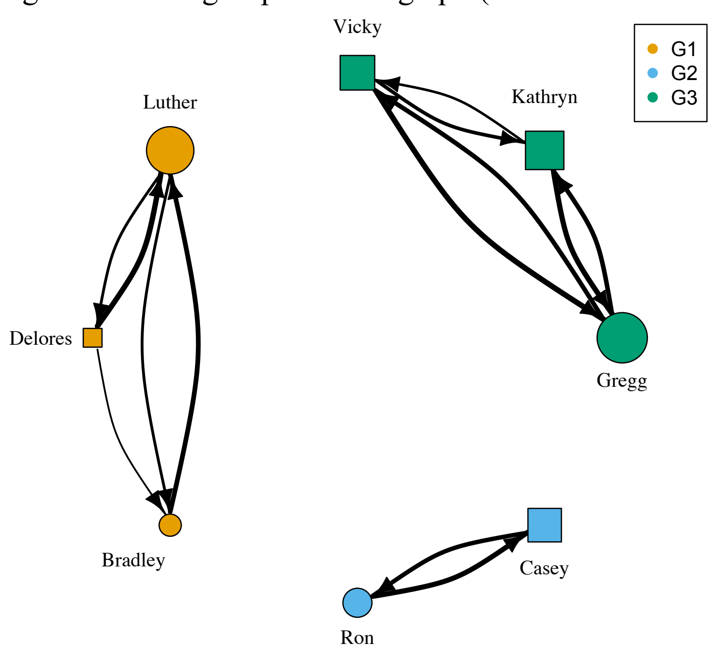


Figure 7. Small group network graph (nominations limited to members of the same group).



This concludes the instructor's initial reflections on the data that were shared with him. After reflecting on these data, the instructor independently decided to make changes to his teaching practices in his future teaching. The next section describes a follow-up interview one year later. This section highlights the instructor's experiences and changes to his teaching practices.

Changes in Instructional Practice

During our initial interview with the instructor, the distribution of student participation and students' perspectives of that participation were a major point of conversation. In particular, the instructor was surprised by the variation in amount and quality of participation across students. In reflecting on student participation patters, the instructor suggested random response strategies as a way to flatten that distribution, though it was not something he had experience using in his own courses.

A year after the original analytics were provided to the instructor we had a follow-up interaction with him in which he enthusiastically described changes to his teaching practice. This prompted us to conduct a follow-up interview. In that interview, the instructor described two methods for random calling on students,

> In whole class discussion I used tongue depressors with each student's name. These were in a small bag and so I could easily grab one out of the bag. The other strategy I used was to decide who from a small group would report back on their group's work. When circulating from group to group I would decide which groups I wanted to share back and when I was at that group I would quickly count off 1-2-3-4 and then roll a four-sided die. Easy!

When asked how the strategies worked in class, the instructor responded as follows,

> I really liked using both strategies. It took the pressure off of me to monitor who I had or had not asked to contribute. It seemed like students liked it too because they knew it was random and so they were not being singled out.

Here the instructor describes the cognitive load (during previous semesters) of trying to keep track of which students had participated to ensure that a variety of students were participating. This is contrasted with the reduced load of letting the random mechanisms choose students. While random calling was added to his repertoire of practice to better distribute participation, he also noticed an additional benefit: it helped bring a joking atmosphere to the class too,

> It was also fun to joke around about this. For example, after grabbing a stick with someone's name on it I occasionally say, "It's not my fault!" and everyone would laugh and the student seemed to feel less pressure.

The instructor also noted challenges to using the methods, because sometimes students did not want to respond after being called on,

> The one thing I struggled with a bit was what to do if a student in whole class wanted to decline. This didn't happen too often but when it did I encouraged them to share their initial thinking, however tentative. Reassure them that we are just getting ideas out, etc. etc.

Here the instructor notes that students were not always comfortable with the random calling, because they were hesitant to share their in-progress thinking. Nevertheless, the instructor encouraged them to share. In addition, the instructor described a consultation with a colleague around this issue,

> Another strategy that I learned from a colleague about half way through the semester is to give the student pass, but then let them know that we will come back to them after the next person talks and they will have to say in their own words the person's idea or provide some other contribution. I liked this approach because it helps to maintain students listening to other students.

Here the instructor describes how the consultation led to a new teaching tool. This highlights one of the ways that EQUIP analytics can support instructional change. First, the instructor reflected on analytics, which prompted a change in teaching practices. Second, this desire to try something new provided impetus for the instructor to consult a colleague about new teaching practices.

In addition, reflection upon the analytics may have made the instructor more aware of which students were participating in the class. In the following response, the instructor expresses disappointment when one of the discussions became dominated by men in the class,

> One [thing] that took me aback was what happened one day when I wanted all five groups to share back (there were 21 students in the class) and I said that they could decide who in their group would share back to the whole class. The first three groups all had a guy report back for them. Ugh. So I kind of made a joke about it and asked the next two groups to diversify, which they did. I think I let them pick one or two other times who would share back to the class but I said something like it can't be all guys reporting back. That seemed to work well enough.

Here the instructor describes an episode in which he did *not* use random calling. He was disappointed to find that three men in the class were all nominated by their groups to share. It is possible that reflection on EQUIP analytics *and* the use of random calling made the instructor more aware of who was participating and what the classroom looked like when all students were contributing more equally. To be sure, random calling is certainly not a panacea, but in reflecting on the future use of these techniques, the instructor responded,

> These two random call strategies are definite keepers.

Although this work is preliminary, we believe that this shows promise for using EQUIP to support instructional change.

Discussion

Mathematics classrooms are complex spaces. While it is known that active learning environments provide opportunities for students to participate in mathematical reasoning, less is known about how opportunities to participate are distributed amongst students. This is an underexplored area in undergraduate mathematics education research. This paper contributes to filling this gap by providing a methodology for looking into participation patterns in an active learning classroom, triangulating teacher perspective, classroom participation, and students' subjective experiences.

There are many ways in which the three perspectives provided a consistent picture of the focal classroom. For instance, the classroom-level statistics were consistent with the teacher's vision of a discourse-focused classroom space, in which norms were strongly established for students to share their thinking and justify their responses (Yackel & Cobb, 1996). These three perspectives also revealed differences of perceptions through the three perspectives. Gregg and Luther were seen as central classroom actors from all three perspectives, but whereas the instructor considered Kathryn to be as equally central as them, the EQUIP data ranked her squarely in third, and the students' surveys put her fourth. Some of these differences could be due to different information considered by each perspective; EQUIP did not focus on small group discussions, and only the instructor could see turned in homework assignments. From a methodological perspective, it is clear that adopting only one perspective may be insufficient to get a clear picture of student participation, given these some of these notable differences in what the different perspectives can describe.

Our study has some key limitations. First, we did not have EQUIP data for student interactions in small groups, which we would like to focus on in future research. Second, the sociograms provided some insight into student experiences, but we would be able to gain a richer picture through follow-up interviews. Third, our sample size was limited and did not allow us to meaningfully focus on issues of gender or race, which we would like to consider in future work. Fourth, this study focuses only on participation, but not actual student outcomes. Finally, although the work is situated in undergraduate mathematics, we did not focus explicitly on content. Nevertheless, we believe questions of who gets to participate and how are different in undergraduate mathematics than classrooms in general, and this is an important area for further study. Despite these limitations, we see important methodological contributions both for the study of participation and for supporting instructor reflections.

A second important contribution is a methodology for engaging instructors with analytics as a way to inform their future teaching practices. When given the data, the instructor found the whole class and individual student data to be consistent with his own vision and experiences teaching the class and was able to see that the norms he strived to support were taken up by the students. At the same time, the instructor also reflected on individual student participation and decided to introduce a new practice into his repertoire: random calling. This prompted the instructor to reflect on his practice and consult colleagues on how to implement the new practices. The instructor largely found the use of this new technique to be beneficial. In addition, the use of this practice helped the instructor open up space in his classroom to talk with students about who was participating (e.g., in his mention of "three guys" being nominated to share).

Although this work is preliminary, in reflecting upon this study, we have learned some key lessons about how to engage in this type of professional development work. First, it requires a high level of vulnerability for an instructor to open their practice and receive these analytics. Especially given the sensitive nature of data describing patterns of participation, it is important to properly frame this work to an instructor, and act as a partner, not an evaluator. Second, it is important to give a clear description to an instructor before doing this work about what results might potentially be found (e.g., that some students did not have as many opportunities to participate). This would ensure that an instructor is prepared and does not potentially feel blindsided by any unexpected data. Third, we feel it is important to provide analytics to an instructor and give them processing time before having a follow-up discussion. This would allow the instructor to more deeply process the feedback before being required to share their thinking with someone else.

Our approach here suggests a possible route forward for working with instructors around student participation. Here we conducted post-hoc analyses on an entire semester of instruction and used it as a tool for reflection with the course instructor. However, like the feedback we give our students, this is too late to be actionable until a future iteration of the course. In the future, we would like to conduct studies where we generate analytics and provide feedback in an ongoing fashion during the semester. Given data in a timelier fashion, an instructor could actually implement these practices and we could study the impact on student participation in response to new practices. We find this a productive avenue for our future research as we continue to study issues of participation in undergraduate mathematics.

Conflict of Interests

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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