A primer on small group instruction in undergraduate mathematics

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Keywords: active learning; group work; STEM education

Abstract: Increasing evidence supports the use of small group instruction to enhance student learning in undergraduate mathematics. Yet, effectively enacting such learning methods requires instructors to develop new skills as educators. This paper provides a practical guide for group work in undergraduate mathematics, describing key aspects of group work through the lenses of forming, norming, storming, and performing. I provide vignettes from my own teaching for reference.

Biography:

Dr. Reinholz specializes in STEM education transformation with the aim of increasing equity and diversity in STEM fields. His work is grounded in a holistic design perspective, which draws on research in disciplinary learning, equity, and organizational change. At the classroom level, he focuses on how reflection and peer feedback can deepen disciplinary learning. Beyond the classroom, he studies how cultural and structural features of higher education can support and inhibit meaningful transformation. Through his leadership in the Access Network, he works to improve access to meaningful STEM learning across the US.

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Introduction

The value of active learning is supported by numerous empirical studies (e.g., Freeman et al., 2014; Kogan & Laursen, 2014) and is also now recognized by key mathematical societies in the US (Braun, Bremser, Duval, Lockwood, & White, 2017; CBMS 2016; Saxe et al., 2015). Consider the following student reflection from an active mathematics classroom:

> I like how it forces everyone to work together. You get multiple perspectives from other people, you see the same problem but you look at it differently. You see better ways to demonstrate and display your results and stuff.

Another student described the benefits of peer support:

> In other classes, sometimes I was afraid to ask questions of professors because l felt embarrassed. However in this class, l have my group members who are willing to help and share their ideas.

These quotes highlight the potential learning benefits when students engage with one another to develop mathematical ideas. In particular, group work is a common method for implementing active learning. In addition to supporting content learning, group work helps students develop a wide variety of collaborative learning skills (Bossert, 1988), which can support their successful futures in the workplace.

As an introduction to group work, this paper provides a modest synthesis of the literature. This synthesis is organized around four themes: forming, storming, norming, and performing (Tuckman & Jensen, 1977). It begins by providing advice for *forming* groups, the process of *norming* groups, how to address inequity and conflict (*storming*), and it provides further resources for instructors who have groups that are *performing*. I illustrate these ideas with vignettes from my own teaching.

Theoretical Background

What is active learning?

Active learning is defined by the Conference Board of the Mathematical Sciences as follows (CBMS, 2016):

> Active learning [refers] to classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking.

From this perspective, active learning is a set of activities that support students to make sense of mathematics (Braun et al., 2017). Active learning provides opportunities for students to build on what they already know (Smith, diSessa, & Roschelle, 1993) and to develop mathematical practices by authentically engaging with the discipline (Engle, 2012; Lave, 1996). Like learning to ride a bike, play a musical instrument, or bake a cake, active learning provides opportunities for students to do the things that they are supposed to learn. While short, informative descriptions (or lectures) can still be valuable, they alone they are insufficient for developing deep understanding. When direct instruction is accompanied with appropriate active learning techniques in the classroom, it has a significant impact on student success (Freeman et al., 2014).

Some elements of successful group work

This paper focuses on group work, a specific type of active learning. Like other active learning techniques, group work generally promotes better student outcomes (Springer, Stanne, & Donovan, 1999). Thus, the question is not whether or not group work can support students, but under what circumstances group can work be made most effective. Two review studies from K-12 education provide a useful starting point. The first review, by Slavin, found two key elements to successful group work (Slavin, 1996):

1. *Group goals* that require the learning of all group members to be achieved; and
2. *Individual success* is consequential to the success of the group as a whole.

Taken together, these factors provide students with incentives to help each other and to ensure that all group members are successful. In contrast, when students are rewarded for a product that could have in theory been created by only one group member, this positive interdependence is no longer required, and the positive impacts of group work diminish (Slavin, 1996).

Johnson and Johnson have also spent decades studying what makes effective groups (Johnson & Johnson, 1991). Their summary has five key characteristics (Johnson, Johnson, & Smith, 1998):

1. *Positive interdependence.* Students perceive their success as dependent on the success of the group as a whole.
2. *Individual accountability*. Students must use what they learn as a group to perform better as an individual.

3. *Positive interactions*. Students build meaningful, lasting relationships.
4. *Social skills*. Students are taught explicitly how to work together.
5. *Group processing*. Students reflect on their group experiences to improve their collaboration.

As with Slavin's (1996) analysis, these five characteristics focus on *creating conditions where the success of each individual group member depends on the success of the group*. To create such conditions, instructors must provide meaningful learning activities and provide sufficient support for student collaborations. How one creates such conditions depends on the setting they are teaching in and their individual style as an instructor. There is no one-size-fits-all, but the following research-based suggestions provide a productive starting point. The first step is determining how to form groups.

Forming

The foundation for group work is the groups themselves. This section addresses three issues related to forming groups: group size, group composition, and group stability.

Group Size

The appropriate size for a group depends on the task at hand. Some tasks are best suited for individual students (e.g., procedural computations). A *group worthy* task is open-ended, requires non-algorithmic thinking, and allows for rich engagement (Lotan, 2003). It is a problem that is appropriate for many minds. By choosing tasks that require multiple minds to be solved, an instructor can help create positive interdependence in the group, because no single member can complete the task individually.

A group of two students (a dyad) is the simplest group configuration. A dyad provides a setting in which all students are positioned to contribute to the group because they are interacting with just one peer. This makes it difficult for students to stay on the margins of the group. Dyads are particularly useful when students are able to compare and contrast their different approaches to a problem, which deepens conceptual learning (Damon, 1984; Reinholz, 2015). However, dyads may not be appropriate for more difficult tasks.

Three to five students in a group is the most common group configuration (cf. Beichner et al., 2007; Cohen & Lotan, 1997, 2014). With three students in a group, all students are more likely to remain engaged (Heller & Hollabaugh, 1992). As group size increases, it is likely that more students will be disengaged, especially if the tasks at hand can be completed by only a few students working together. The fundamental tradeoff in group size is between students being on task and having more minds to engage with a specific task. In my own teaching, I favor groups of three, to maximize the opportunities for each student to participate.

Group Composition

Research shows that students are more likely to have a positive experience in instructor-chosen teams than student-chosen teams (Layton, Loughry, Ohland, & Ricco, 2010; Oakley et al., 2004). Self-selection can lead to a lack of diversity, groupthink, and excessive homogeneity, which may further marginalize certain students. For these reasons, I recommend that instructors assign groups, with the assistance of students.

Research shows that when groups are gender-balanced, or have a majority of women, the women in the group are more likely to participate and have reduced anxiety (Dasgupta, Scircle, & Hunsinger, 2015). In contrast, groups dominated by men can reduce women’s sense of belonging and reinforce negative stereotypes about women in mathematics, which results in worse outcomes for women (Grover, Ito, & Park, 2017). Research also shows that emergent bilingual students (sometimes referred to as English Language Learners) can benefit from having friends in their groups. This allows them to engage in a larger variety of roles, rather than just getting helped by peers (Takeuchi, 2016). Thus, if instructors are working with emergent bilingual students, they may take care to consider placing them in groups that they would be more comfortable in with their peers.

The research with regards to race and group formation is less clear (cf. Esmonde, 2009). Nevertheless, research highlights the negative impact of racial stereotypes that position some groups as mathematically capable and others as less capable (e.g., the "white male math myth," Stinson, 2008; or "Asians are good at math," Shah, 2017). For this reason, instructors should be aware that issues of status and race will be at play in small group interactions, and they may not be easily mitigated in group formation (Brown & Mistry, 1994). Thus, an instructor must attend to them as a part of facilitating group work (see below in *storming*).

Given the complexities of attending to equity with regard to race, gender, and individual personalities, there is no perfect recipe group composition (although some are better than others). One proposed solution is to group students by "ability." However, such an approach is problematic for a number of reasons. First, it is difficult to accurately assess what students know and what their potential for learning may be. Second, grouping by ability reinforces the idea of "good" and "bad" students, which tends to result in certain students (sometimes unintentionally) receiving worse instruction (Louie, 2017). Finally, despite whatever intuitive appeal such an approach has, empirical evidence shows that it increases inequity (Gamoran, 1992). For all of these reasons, I strongly advise against ability grouping.

Group Stability

When students remain in a static group for extended periods of time, it allows them to form deep connections with their peers, supporting positive interdependence (e.g., Fullilove & Treisman, 1990; Michaelsen et al., 2008). When static groups are used, it is important for group assignments to result in productive groups, so that some students are not stuck in a low-functioning group. This can be achieved by using a variety of random and flexible groupings early in the semester, before developing more stable groups. This allows the instructor to get more information about the students and how they work together. After these early flexible groupings, an instructor can also ask all students to identify peers that they would work well with, and use this information to form static groups. This approach can help ensure that the most vulnerable students in the classroom are supported.

A complementary strategy is to use semi-permanent groups. For instance, every 4-6 weeks an instructor can check in with the students about how their groups are working. If a group is not working out, it can be given the opportunity to disband (Oakley et al., 2004). This way, high-functioning groups can remain, and an instructor can reassign students from the low-functioning groups. If only one group chooses to disband, their members can be added to other groups (e.g., a group of three can be disbanded and added to other groups of three to create three groups of four). Once groups are formed, the next step is to help set the standards for how they will engage. This is described in the next section, norming.

Norming

In some classrooms, students begin each day by working on a warmup problem. In others, they know that answers are supposed to be put in boxes. In some classrooms, students only ask questions to their instructor, but never talk to their peers. In others, they make bold mathematical claims. How do students know which behavior is appropriate? It comes down to the norms that an instructor sets for the classroom.

Norms are regular patterns of interaction that regulate how individuals within a classroom should interact with each other (Yackel & Rasmussen, 2002). These norms may be social (e.g., explain your thinking, do not just provide answers), or sociomathematical (e.g., what constitutes an acceptable mathematical justification; cf. Yackel & Cobb, 1996). These norms help a classroom function more effectively, and also bring students into the culture of a discipline. Thus, in order for a classroom to operate in the desired fashion, an appropriate classroom culture must be created that leads to the successful uptake of these norms. For productive group work, an instructor wants to create positive interdependence between the students. In the following sections, I provide vignettes from my own classroom practice and give examples of practical techniques.

Establishing Norms

The most basic failure mode for group work is that an instructor never works with the students to establish norms and instead just "gets right to work" in teaching content. It is often assumed that because students are in college that they will already know how to productively work together in groups. Even if students may have had productive collaborative experiences, they are almost certainly not uniform, and may not be specific to the discipline of mathematics. Thus, without guidance, it is unlikely that group work will proceed in an effective way, or at the very least, it will not proceed as the instructor may hope for. In contrast, when an instructor sets aside space in the classroom to talk about group processes, it signals to the students that process is important. Basic norming conversations may only take 10-20 minutes and set the classroom up to be much more productive.

One way that I like to establish norms in my classrooms is through a “looks like sounds like” chart (Hoffer, 2012). To begin, I tell students that in this class we will be working together in groups to develop mathematical ideas and it is important that we decide as a community how we would like to structure our collective work. I tell the students that we are going to discuss what good group work “looks like” and “sounds like.” I give the students a few minutes to brainstorm in their groups, and then we come together to discuss as a class. The first student raises their hand and offers “all students are actively involved in the work,” which I put under “looks like.” Another student says “asking questions when you’re not sure,” and another student says “listening to your group members,” both of which I put under “sounds like.” A fourth student offers “not being on your cell phone” as another case of what good group work “looks like.” I ask the class what they might do if they notice one of their group members is not participating, and they offer “asking questions and checking in with quiet members” as a “sounds like” example. We continue this process until the class is satisfied with our list. Whenever there are ideas that I would hope to get out in the discussion that the students have not yet suggested, I ask leading questions, like the one above.

The list is as a starting point; I will make a copy for each student and also occasionally project it at the beginning of class. The class may modify our list as the semester goes on. There is nothing special about using this type of chart; it is merely one of many methods for talking about group processes.

An alternative method for establishing norms is a *paper toss*. In a paper toss, each student writes up an important idea for collaboration on paper, crumples it up, and throws it across the room (Keeley & Tobey, 2011). Then each student finds a new piece of paper and shares what they found. This lowers barriers for different students sharing their ideas, because the risk inherent in sharing their own idea is taken away.

Norms can also be established using sample activities that focus on process (e.g., Cohen & Lotan, 2014). An instructor can also use a regular task designed for the course, and incorporate a 10-20 minute debrief conversation after the task is completed. The instructor can ask what worked well and what was challenging and how these ideas might inform good collaboration on future activities. This embeds the norming conversation in the actual work of the class. Regardless of the method used, when an instructor creates space for talking about process, it communicates to students that process matters. If the instructor does not make time, it communicates a different value.

A final approach is to use *team contracts* (Michaelsen et al., 2008). To create a contract, each group sets standards for what each group member must do to contribute to the group. This could contain ideas such as: “inform group members if you will be absent from class,” “be prepared for class by completing the pre-reading,” or “make space for all group members to contribute.” Team contracts allow each group to set its operating parameters, and they can also provide fodder for a whole-class discussion. Students should be given opportunities to revise their contracts later in the semester, as they realize what is required for their groups to work well.

Promoting Norms

Establishing norms in no way guarantees that students will follow them. If norms are set once at the beginning of the course and never referred to again, they are likely to be ignored. As such, an instructor needs to remind students of the norms set and promote them as a part of classroom interactions.

In my classes, I use a clipboard to monitor student interactions in small groups. I use a "+/quote/-" chart to help support students to achieve productive engagement (Hoffer, 2012). While students are working together, I make notes on the clipboard, documenting productive interactions (the plus), unproductive interactions (the minus), and interesting quotes from students (the quote column). Examples of a "plus" I would document include, "all students working together on a problem," "students ask each other questions," or "checking answers to make sure they make sense." In the quotes, I note student statements I would like to share with the class, such as "Amelia, your strategy to use the contrapositive here was a really great idea!" In the "minus" section, I note things that would break our class norms, such as "using a cellphone," or "not listening to other group members."

After group work time, I project this chart and discuss process with students. In general, I praise students publicly (i.e. highlighting good things by name) and criticize privately (i.e. not calling out disruptive students in front of the whole class). Thus, I would generally attribute names and groups to the positive things and make more general statements about the negative, such as "some of the groups on this side of the room." Having the clipboard to monitor student engagement and having a short debrief (about five minutes) communicates again the importance of process. It also provides a way to show students what you care about, because you can draw attention to social and sociomathematical practices that you would like to foster. Finally, these discussions can be used to provide space for students to share their own experiences working in groups, as they may like to highlight something that worked really well for them. This is also a way to emphasize the importance of participation, especially if participation is part of the grading scheme in the course.

Promoting norms also happens in how a class is facilitated. Especially in introductory courses, like calculus, I try to teach my students that explaining their ideas, not just giving answers, is a key part of mathematics. To begin, this means that I must ask questions that require students to explain, not just give answers. However, at the beginning of the semester, students will almost certainly just provide an answer rather than giving an explanation. If I were to simply accept the answer, I would be negating this norm. Instead, I follow-up with the students until they explain their ideas and also emphasize to the class that explaining is part of the work we are doing in the class. A common way that I have seen instructors undermine this norm is by answering their own questions, which soon teaches students that they do not need to answer the questions, because their instructor will do it for them.

Group roles are another way to promote classroom norms (Cohen & Lotan, 1997; Michaelsen et al., 2008; Moog & Spencer, 2008). Examples of roles include facilitator, manager, recorder, or skeptic. For example, a facilitator would be responsible for making sure that all group members have a chance to participate, and a skeptic would make sure that the group justifies their answers. Roles describe a particular aspect of effective group process, and are rotated across different group members each day. This allows an instructor to draw attention to particular processes, and again, the instructor can include time for debrief activities related to the use of roles. While I have not often used roles in my own personal teaching experiences, I draw attention to them here as a possible technique, because they are widespread technique.

Accountability to Norms

Norms can also be enforced through accountability structures. The most common way to do so is to have some sort of grading system that relates to group participation. One way to do this is have students (privately) rate the contributions of their peers in their group, which ultimately affects the grades of other group members. This participation grade can be a modest amount of the overall course grade (5-10%). Other grading schemes make participation required to pass a course, so that if students miss too many classes or do not participate they would not be able to pass. One can also use both individual and group quizzes, or even allow students to improve their grade in part based on the grades of peers in their group (Slavin, 1996). Thus, there are a variety of options for including participation in a grading scheme.

Storming

While group interactions can support deep learning, without conscious effort, they tend to be inequitable (Esmonde, 2009). Because opportunities to participate are opportunities to learn, it is important that all students have an opportunity to participate meaningfully. The most common issue is when a particular student dominates a group, or conversely, when certain students have limited opportunities to participate. I introduce three instructional techniques to help address these issues.

Imbalances in group participation are generally the result of status imbalances. In particular, high-status students (e.g., who are perceived as smart or are popular), tend to dominate conversations and prevent other students from participating equitably (Langer-Osuna, 2016). These perceptions of status are often linked to racialized and gendered narratives about who can do math (e.g., Shah, 2017; Stinson, 2008) and are not reflective of students' actual abilities. Thus, while an instructor can mitigate status imbalances and open up broader opportunities for participation in their individual classroom, it is only a small step towards addressing larger systemic issues that produced the inequities in the first place.

Complex Instruction offers concrete techniques for addressing status issues (Cohen & Lotan, 2014). The first technique is the multiple ability treatment, which states that group tasks should be framed as requiring multiple abilities (e.g., organization, communication, explaining, analysis), and that not all students will be experts at all of them, but each student is an expert with at least some of them. This helps create positive interdependence. The second technique is called *assigning competence*. When a low status student makes a useful contribution, the instructor publicly draws attention to it. For instance, a quiet student may share a partially correct idea about how to solve a problem. By drawing attention to this (e.g., in a whole class discussion), and highlighting the positive impact of the students' idea, it can help elevate the status of that student and improve their engagement in the future.

To assign competence to students, an instructor must first make sure their ideas are brought out in group discussions. One way to do this is by addressing questions to specific students in the group (e.g., can you explain what your group has done so far?), especially those who are low status. By addressing questions to specific students, it makes sure that a single student cannot dominate the group and always act as a spokesperson for the group. If questions are addressed to the group as a whole, it is likely that one or two students will always answer them.

The five practices for orchestrating discussions (anticipating, monitoring, selecting, sequencing, and connecting; cf. Stein, Engle, Smith, & Hughes, 2008) are another powerful set of techniques for addressing equity in group work. During work time, an instructor monitors and selects ideas to be shared later during the synthesizing discussion. This allows an instructor to give students advance notice that they will be asked to share, which broadens opportunities for participation. It also provides a concrete way to assign competence, by drawing out student contributions from groups to publicly sequence and connect them to the discussion in a coherent way.

A third technique is the use of wait time (Rowe, 1986). Wait time refers to the time between asking a question and calling on a student to respond. When an instructor always calls on the first student to raise their hand, it promotes a classroom environment that is dominated by high status students. In contrast, when an instructor introduces wait time (e.g., by counting silently to 5 in their head), more students are able to contribute. Similarly, an instructor can use another form of wait time: intentionally pausing *after* a student makes a contribution. This draws attention to that particular contribution, and thus can help raise the status of a student.

All of the above methods are subtle techniques to help create an inclusive classroom, but they do not address conflict situations directly. Inevitably, conflicts do arise when people are working together. Such conflicts can be handled by speaking with students individually outside of class time. Ultimately, if students are not working well together, the instructor can use opportunities to change groups to remedy the situation. Still, if the behavior of a particular student is problematic towards other students, it needs to be addressed directly, or even after groups are changed the issue may surface again.

Performing

This primer has provided an overview of how to form and norm groups, and deal with storms that inevitably arise. Because the literature on group work is vast, providing a summary of all of the possible techniques is beyond the scope of this article. Instead, I provide a set of further resources for interested readers, who may already have groups that are beginning to perform.

The first set of resources relates to “branded” teaching techniques. These approaches are holistic, insofar that they have a underlying philosophy and a suite of teaching techniques to achieve a desired goal. Some well-known techniques include: Complex Instruction (Cohen & Lotan, 1997), Process-Oriented Guided Inquiry Learning (Moog & Spencer, 2008), Team-Based Learning (Michaelsen et al., 2008), and Modeling Instruction (Brewe, 2008). These techniques span the disciplines of mathematics, chemistry, medicine, and physics, respectively. All of these techniques center around group work, but they all have different tools for establishing norms and organizing collaboration.

Another resource is the MAA’s *Instructional Practices Guide* (MAA, 2018). This extensive guide provides concrete teaching techniques, vignettes, and explanations to support active learning in mathematics. Readers might also consider publications from the National Council of Teachers of Mathematics, such as *Taking Action* (Boston, Dillon, & Smith, 2017). Other online communities of interest include *Art of Mathematics* (https://www.artofmathematics.org/) and the *Academy of Inquiry Based Learning* (http://www.inquirybasedlearning.org/). These communities can serve as a starting point for connecting with others who are also using similar teaching methods. A summary of methods described in this paper (a reference sheet) is given in Table 1.

Table 1. Reference sheet.

	Issues	Advice / Techniques
Forming	Group Size	2-4 students
	Group Composition	Instructor assigned heterogeneous groups, with attention to race, gender, and individual needs
	Group Stability	Semi-permanent groups, with opportunities for adjustment
Norming	Establishing Norms	Norms conversations, “looks like, sounds like,” paper toss, group-process activities
	Promoting Norms	Monitoring interactions, facilitation, group roles
	Accountability to Norms	Peer ratings, participation-based grading, individual/group quizzes
Storming	Inequitable Participation	Multiple ability treatment, assigning competence, five practices, wait time
Performing	Further Reading	Complex Instruction, POGIL, TBL, Modeling Instruction, MAA IP Guide, NCTM Publications, Art of Mathematics, AIBL

Summary

The evidence is clear: group work has numerous learning benefits for students. Moreover, working with others is required in the modern workplace. This paper offers a modest synthesis of literature in K-12 and higher education to guide undergraduate mathematics educators. Using mutual interdependence as a foundation, instructors can use a variety of methods to support productive collaboration. These methods help an instructor form groups, establish norms, address storms, and seek further resources as the groups begin to perform.

This article has its limitations. It provides an incomplete synthesis of the literature on group work, as it would be impossible to provide an exhaustive description of work in the area. Moreover, each classroom is unique and there is no one-size-fits-all. There may also be larger systemic issues that are hard to address, like racism and sexism, or teaching using collaborative methods while embedded in a culture of lecture-based classrooms. Finally, much of learning to teach using group work requires practice, so while this manuscript provides guidance, instructors will need to develop their skills to effectively promote group work.

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