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Co-Calculus: Integrating the Academic and the Social

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Abstract

Being part of a cohesive learning community supports retention and success in early mathematics courses. Yet, large, unwelcoming lectures stand in opposition to this goal, isolating students and pushing them away from STEM. This paper offers a comparative analysis of three efforts to build community amongst students, all situated within a single large-lecture introductory calculus course at a diverse, research-extensive institution. These programs were: (1) active learning labs, targeted at all students, (2) a small-group seminar for commuter students, and (3) a workshop model targeted at "underserved" students in STEM. Of these three efforts, only the workshop model had a significant impact on student success. Social networks within the workshop section suggest that students were able to integrate their academic and social experiences to a greater extent than the other sections. These results suggest that active learning and co-calculus experiences alone may be insufficient to foster cohesive social and academic bonds, unless properly organized.

Introduction

Who can be a mathematician, scientist, or engineer? In an ideal world, all students would have equal opportunities. In reality, educational inequity remains a major concern (National Academy of Sciences, 2007; President's Council of Advisors on Science and Technology, 2012). In particular, the Science, Technology, Engineering, and Mathematics (STEM) professionals of tomorrow are greatly influenced by *who* the STEM professionals of today are. For instance, students with parents who attended college have an easier time in college than those whose parents did not (Ishitani, 2003; Stephens, Fryberg, Markus, Johnson, & Covarrubias, 2012; Stephens, Townsend, Markus, & Phillips, 2012). It is not just a matter of professional knowledge, but of access to cultural ways of knowing (Stephens, Townsend, et al., 2012). It is about being in a community with others who look and speak like you. It is a matter of belonging.

Indeed, students often abandon STEM career aspirations due to unwelcoming and uninspiring environments, *not* lack of ability (Seymour & Hewitt, 1997). Across domains, a sense of belonging is linked to success (Good, Rattan, & Dweck, 2012; Lewis, Stout, Pollock, Finkelstein, & Ito, 2016). This is particularly important in mathematics, as introductory calculus courses significantly decrease student confidence, enjoyment, and interest in mathematics (Bressoud, Carlson, Mesa, & Rasmussen, 2013). While these effects impact all students, they differentially impact non-dominant students. For instance, female students with the same grades as their male counterparts are 1.5 times as likely to leave the calculus sequence (Ellis, Fosdick, & Rasmussen, 2016).

Changes in instruction can help. For instance, active learning improves the success of all students (Freeman et al., 2014), and can improve persistence in later courses too (Kogan & Laursen, 2014). Despite this promise, most college mathematics classrooms in the US are still dominated by instructor-centered teaching (Apkarian et al., 2016; Lutzer, Rodi, Kirkman, & Maxwell, 2005). Given the challenges of enacting educational change (Austin, 2011; Fairweather, 2008; Kezar, 2011), and sustaining such changes (Dancy & Henderson, 2010; Henderson & Dancy, 2007; Lutzer et al., 2005), large lecture calculus courses are unlikely to disappear in the short term.

As a complementary approach, co-curricular programs are growing in popularity. For example, many US institutions now feature first-year experience that integrate students into university life through a combination of first-year courses and learning communities (Schmidt & Graziano, 2016). Such approaches show promise for non-dominant students in STEM fields such as biology (Graham, Frederick, Byars-Winston, Hunter, & Handelsman, 2013) and physics (Albanna, Corbo, Dounas-Frazer, Little, & Zaniewski, 2013; Gandhi, Livezey, Zaniewski, Reinholz, & Dounas-Frazer, 2016). In mathematics, the situation is somewhat different. While some institutions offer a "co-calculus" learning experience for students, these opportunities are not widespread; for instance, only 10 of 222 US institutions surveyed for the Progress through Calculus project offered co-calculus

(Apkarian, Kirin, & The Progress through Calculus Team, 2017). Nevertheless, there is strong evidence in favor of such approaches, from other STEM disciplines and specifically in mathematics (Fullilove & Treisman, 1990). For that reason, they are the focus of this paper.

While their details may differ (e.g., frequency of meetings, content focus), co-calculus sections generally aim to build collaborative student communities, with the goal of having students work together in the target course and ideally in future courses too. The rationale is that building community around mathematical practice helps students integrate the "academic" and "social" dimensions of their lives (Fullilove & Treisman, 1990; Tinto, 1997, 2006). Ultimately, such programs can build community and help students develop a sense of belonging, which is key to persistence and success in STEM (Lewis et al., 2016).

This paper compares three efforts to build community amongst students, all situated within a single large-lecture introductory calculus course at a diverse, research-extensive institution. These programs were: (1) active learning labs, targeted at all students, (2) a co-calculus seminar for commuter students, and (3) a co-calculus workshop targeted at "underserved" students in STEM. This paper explores why the Workshop section was more successful than the other efforts.

Theoretical Framing

When students "perceive that they are valued, accepted, and legitimate members in their academic domain" they are more likely to succeed (Lewis et al., 2016, p. 2). Accordingly, issues of identity and belonging have received considerable attention in mathematics education research (Boaler & Greeno, 2000; Esmonde, 2009; Nasir, 2002). This literature highlights that whether and how students identify with the discipline and with others in the discipline cannot be ignored. Simultaneously, research suggests that particular groups of undergraduate students are less likely to identify with mathematics than others, because their experiences are gendered, racialized, or otherwise different from "dominant" students (Rodd & Bartholomew, 2006; Solomon, 2007). These systemic issues contribute to a lack of diversity in STEM (NSF, 2013).

How can first-year experiences help? The persistence framework emphasizes how authentic disciplinary engagement promotes meaningful learning *and* identification (Graham et al., 2013). First-year experiences do not provide otherwise "deficient" students with remediation. Rather, they challenge students with high expectations and high support (G. L. Cohen, Steele, & Ross, 1999; Steele, 1992), which helps counteract institutionalized racism, sexism, and other forms of marginalization. Indeed, an extensive literature documents that students of color must overcome institutional barriers that their white peers do not (Carter, Locks, & Winkle-Wagner, 2013).

In undergraduate mathematics, the Mathematics Workshop Program (MWP) is one of the most well-known co-calculus programs (Treisman, 1992). In his dissertation, Uri Treisman followed two groups of Berkeley students: 20 Asian Americans and 20 African Americans. The African American students mostly studied alone, with only 2 of 20 reporting that they worked in groups and spent about 8 hours on assignments per week. Effectively, they separated academics from social endeavors. In contrast, the Asian American students worked with their peers, and devoted about 14 hours per week to studying. Moreover, they combined their social and academic activities: they spent lots of time working together to study, complete problem sets, prepare for exams, etc. It was conjectured that these differences may account for why the Asian American students at Berkeley were more successful than their African American counterparts.

Building on this conjecture, the MWP was created. This honors program recruited first-year students of all races, but in practice had participants that were 80% African American and Latinx. Students in the MWP had significantly higher passage rates in calculus compared to peers who were not (Fullilove & Treisman, 1990). Students in the MWP met twice a week for 2 hours at a time and worked collaboratively in small groups on difficult calculus problems (Fullilove & Treisman, 1990). Since its inception, the MWP has been modified and implemented at a variety of other institutions across the US (Hsu, Murphy, Treisman, Carlson, & Rasmussen, 2008). The MWP has two key features: (1) it is an honors program, not remediation, and (2) it builds a strong student community (Fullilove & Treisman, 1990). By working on challenging mathematics together, students productively integrate their academic and social experiences (Tinto, 1997, 2006). In contrast, students in remedial settings are less likely to identify with the discipline (Larnell, Boston, & Bragelman, 2014; Nasir & Shah, 2011). While remedial co-calculus offerings increase time on task, they may have a negative psychological impact on students, limiting their impact.

To study student communities, this paper draws on a set of graph-theoretic techniques called Social Network Analysis (SNA), developed for precisely this purpose (e.g., Daly, 2010; Quardokus & Henderson, 2014). SNA has a long history in higher education (Biancani & McFarland, 2013), used to study student communities, for instance in physics (Brewe, Kramer, & Sawtelle, 2012), public administration (Chen, Wang, & Song, 2013) and mathematics (Alcock, Hernandez-Martinez, & Godwin Patel, 2016). A key finding is that mandated formal groupings do not necessarily result in the formation of collaborative relationships (Rienties, Héliot, & Jindal-Snape, 2013); integrating the academic and social evidently requires more. Accordingly, this paper addresses two main research questions:

1. How was student success related to their participation (or lack thereof) in various co-calculus programs?
2. To what extent did these programs support students to integrate their academic and social experiences?

Method

Context

The study took place in calculus I at a relatively large (over 30,000 students), racially diverse (e.g., ~65% students of color) research-extensive university. The course was taught through a combination of large lectures (100-200 students) taught by full-time instructors and smaller breakout recitation sections (30-40 students) taught by Graduate Teaching Assistants (GTAs). Lectures met three times weekly, and recitations met twice weekly, all for 50 minutes. Sections had common exams, homework assignments, and recitation activities. The lecture emphasized student interaction, through: personal whiteboards, classroom response systems, group work time, student presentations, classroom discussion, and Peer-Assisted Reflection (Reinholz, 2015). Recitations also focused on student interaction, mostly consisting of collaborative group work. Recitations meeting twice weekly was a departmental initiative to build community amongst students. Moreover, drop-in support (through office hours) was held in a collaborative workspace on campus, aimed to support student collaboration. The idea was that students would engage in mathematical sense making together on rich mathematics problems.

Four recitation sections (Standard A, Standard B, Seminar, and Workshop) were embedded in a single large-lecture section (N=124 after 2 withdrawals). “Standard” students experienced calculus as described above, while the Seminar and Workshop students also had co-calculus experiences. The two Standard sections were taught by a departmental “lead” GTA, one of the most experience in the department. The Seminar and Workshop sections were taught by a GTA who was teaching calculus I for the first time. Office hours for these sections were all held in the same collaborative workspace on campus.

The university-sponsored commuter success program (the Seminar) aimed for students: (1) to build relationships with peers, faculty, and staff, (2) feel a sense of belonging on campus, (3) join a community of students with common academic and social interests, and (4) receive academic support in a single course that they can later transfer to other courses. This was to support commuters, who tend to be socially isolated on the campus. While the program has various components, most relevant is the co-calculus course that met twice per week for 50 minutes at a time. This course was taught by an advanced undergraduate student from a teacher credentialing program.

Like the MWP, the Workshop program aims to serve traditionally underserved students in STEM (e.g., based on income, first-generation status). Like the Seminar, the Workshop had similar goals to (1)-(4) above, and also focused on industry/alumni engagement. In addition, the Workshop program has a wide variety of professional development opportunities (e.g., internships) for students beyond their first year. The component of focus here is its co-calculus program, which met for 2 hours at a time, twice a week, just like the original MWP. To keep the size of the cohorts small, there were two of these Workshop sections associated with the same recitation, each facilitated by an advanced undergraduate affiliated with the Workshop program. Like the MWP, the Workshop program was framed as an honors program (not remediation), which required students to apply to it. All programs are compared in Table 1.

Table 1. Comparison of student groups in the study.

Sections	Standard A & B	Seminar	Workshop
Features	Lecture: 50 min 3 days/wk Recitation: 50 min 2 days /wk	Lecture: 50 min 3 days/wk Recitation: 50 min 2 days /wk Co-calculus: 50 min 2 days/wk Purpose: support commuters	Lecture: 50 min 3 days/wk Recitation: 50 min 2 days /wk Co-calculus: 2 hrs 2 days/wk Purpose: honors engineering for underserved students

Data Collection

Student grades on all course assignments were collected, and 109 of 124 students consented for the surveys and for their demographic information (e.g., race, gender, major). To study the student communities, social network analysis (SNA) was utilized. Identical surveys were administered to students after the second midterm (near the middle of the semester) and just before the final (at the end of the semester). There were response rates of 69 to the mid-semester survey (63%) and 70 to the end-of-semester survey (64.2%). Each question on the survey had the following stem: *The following questions ask about your interactions with peers in this course. For each question, write up to six names of your peers, including last names when possible.*

1. Who do you work with in class (e.g., on PAR, in groups)? This could be in lecture or activity sessions.
2. Who do you work with outside of class (e.g., on homework, in a study group, in the Math Learning Center)?
3. Who do you consider your friends in the class?
4. Who do you feel has made valuable contributions to our discussions in class (in lecture or activity sessions)?

After each of the questions there were six numbered spaces for students to write the names of their peers. Students were told that they could write the names of peers in other lecture sections of the course, but in practice there were so few responses that included students in these other sections that they were dropped. Also, there were some instances in which it was not possible to identify the target student listed on the survey, because students only wrote the first name of their peer and multiple students had the same first name. In these instances, the students were not included in the dataset. The number of such instances was minimal.

Participant Information

Tables 2-5 provide student demographics. Table 2 shows that the co-calculus sections had many more male students than female students, whereas the standard sections were more balanced. The Workshop section had the most racial diversity (see Table 3), with students in all categories other than White.

Table 2. Gender by section.

	Standard A	Standard B	Seminar	Workshop
Female	14	15	7	7
Male	17	19	26	19
Total	**31**	**34**	**33**	**26**

Table 3. Race by section.

	Standard A	Standard B	Seminar	Workshop
African American	2	0	0	2
Asian/Pacific Islander	3	5	6	7
Hispanic	3	9	8	12
International	1	0	0	1
Multiple Ethnicities	6	0	3	2
White	9	12	12	0
UNKNOWN	7	8	4	2

Table 4 provides end-of-semester GPA as a proxy for student preparation. Comparing the Seminar section to the others in terms of GPA was not significant, $t(N = 124, df = 38.966) = -0.440$, $p = 0.626$. The results were the same for the Workshop section compared to other sections, $t(N = 124, df = 31.177) = 0.025$, $p = 0.980$. As such, it was determined that differences in student outcomes could not be attributed to prior academic preparation.

Table 4. Total GPA by section.

	Standard A	Standard B	Seminar	Workshop
Mean	3.09	3.15	3.03	3.09
SD	0.49	0.43	0.88	0.79

Table 5 shows co-calculus enrollment. All Workshop students received co-calculus, whereas only half of the Seminar students did.

Table 5. Co-calculus programs by section

	Standard A	Standard B	Seminar	Workshop
Yes	-	-	17	26
No	31	34	16	-

Social Network Analysis

A brief review of SNA is provided for readers who may be unfamiliar with the techniques. A network consists of nodes (vertices in graph theory) and relationships (edges in graph theory). A sample network is shown in Figure 1. Here, nodes A and C have degree 1, node B has degree 2, and node D has degree 0. In this paper, nodes represent individuals within a network, while relationships represent the connections between individuals. Graphs may be directed or undirected. In a directed or asymmetric graph (e.g., who is perceived as a valuable contributor), one can differentiate between in-degree and out-degree. The in-degree refers to the number of edges pointing into a node (i.e. how many people nominate a particular node as a valuable contributor) while the out-degree is the number of edges pointing out of a node (i.e. the number of people a particular node nominates as valuable). The "degree" of a directed graph is the sum of in- and out-degree. In an undirected graph, incoming and outgoing edges are not differentiated, and the total sum of the edges is the degree. Undirected graphs are more common for features such as friendship, where relationships are assumed to be reciprocal.

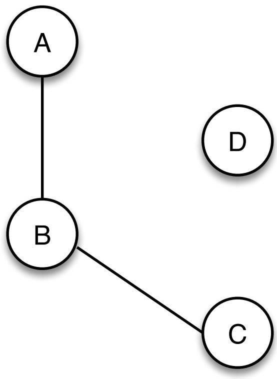

Figure 1. Sample network graph.

Network properties can be described in many ways: this paper uses three. *Density* is one way to describe the general level of cohesion within a graph. Formally, it is the percentage of edges that *do* exist within a network divided by the number of edges that *could* exist. While density is useful for comparing networks of the same size, it can be limited in comparing networks of radically different sizes, because in a larger network the number of potential edges becomes large very quickly. *Centralization* describes the extent to which a graph is concentrated around a few important actors. Centralization is computed by first looking at the centrality of each node, and then taking the ratio of the actual sum of differences to the maximum possible sum of differences. *Transitivity* is a property of triads in a social network. In a network with perfect transitivity, if X is connected to Y, and Y is connected to Z, then X must be connected to Z. In practice, networks almost always have transitivity below one, and the transitivity coefficient describes the probability that transitivity holds in a network. In other words, if X is connected to Y, Y is connected to Z, and the transitivity of the network is 0.25, then there is a 25% chance that X is also connected to Z.

Findings

Student Outcomes

The first research question focused on the impact of the co-calculus sections. Table 6 shows the final exam scores by section. In comparing the Seminar to all other sections, the results were not significant, t(N=124, df=58.521) = -1.1611, p = 0.2503. In comparing the Workshop to all other sections, students scored an average of 10% higher, and the results were significant t(N=124, df=67.293) = 2.7325, p = 0.008022. The effect size was computed used Hedge's g given the small sample size. The results were g = 0.6, which is a medium to large effect size (J. Cohen, 1988).

Table 6. Final exam scores by section

	Standard A	Standard B	Seminar	Workshop	**Overall**
Mean	63.42	59.44	58.79	70.31	62.54
SD	23.32	25.55	21.47	13.83	22.05

Given that there were no differences in the Seminar section compared to the other sections, one may ask whether it was due to only a number of the students receiving co-calculus. To investigate this possibility, Table 7 compares students who had co-calculus with those who did not. The differences were not significant for final exams, t(N = 33, df = 30.372) = 0.201, p = 0.8421, or for GPA t(N = 33, df = 24.319) = 1.752, p = 0.09238.

Table 7. Scores in the Seminar section.

Mean (SD)	Co-Calculus	No Co-Calculus
Final Exam	59.52 (20.92)	58.00 (22.70)
Total GPA	2.80 (1.05)	3.32 (0.51)

Table 8 shows the passage rates by section. Consistent with the impact on student final exam scores, the passage rates in the Workshop section were approximately 10% higher than in the other sections.

Table 8. Passage rates by section.

	Standard A	Standard B	Seminar	Workshop	**Overall**
Pass	26	27	27	24	**104**
Fail	5	7	6	2	**20**
Passage Rate	83.9%	79.4%	81.8%	92.3%	**83.9%**

In sum, the quantitative results show that the students in the Workshop section did significantly better than all of the other sections, and there were no statistically detectable differences in the Seminar section compared to the other sections. We now consider the social networks across sections to better understand why the Workshops may have had this impact.

Social Networks

The following section is broken into three subsections: whole-class networks, individual section networks, and statistical analyses. The whole class networks provide a baseline description of social networks in introductory calculus. This will provide background framing for future studies using SNA in calculus, because they will have a reference to compare to. The individual section networks provide insight into whether the individual sections achieved their goal of supporting students for academic and social integration. This is the second research question in this paper. The final section, with statistical models, explores whether network characteristics could be used to predict students' final exam scores.

Whole-Class Networks

To begin, the properties of the whole-class network are provided in Table 8. Results are given for both the midterm and final surveys to illuminate any possible changes in the network over the course of the semester. Edge density, as given in Table 9, indicates that the in-class and friendship networks had the most connections, followed by out of class, and finally contributions. This means that even though students may work together in class and consider one another friends, that does not necessarily translate into forming out of class collaborations. These are instances with social, but not academic, integration.

In terms of centralization, the final friendship networks were most centralized. This indicates that friendship networks formed more around key actors, as compared to the other networks. As one would expect, the contributions networks also had a high centralization, because they are organized around a few key students. Finally, the transitivity measures indicate that the out of class network had the highest transitivity. This meant that if two dyads were connected, there was almost a 40% chance that the triad was connected. This suggests that students were much more likely to work in groups outside of class, rather than just in pairs.

Table 9. Network properties of the whole-class network.

	Time	In Class	Friendship	Out of Class	Contributions
Edge Density	Midterm	0.023	0.020	0.014	0.006
	Final	0.021	0.022	0.015	0.006
Centralization	Midterm	0.047	0.051	0.065	0.078

	Final	0.049	0.104	0.056	0.085
Transitivity	Midterm	0.276	0.309	0.302	0.142
	Final	0.271	0.290	0.385	0.145

Figures 2-5 show the entire network for the whole class. The following analysis focuses on the final networks. For all graphs in this paper, male students are indicated with circles, while female students are indicated with squares. The size of a given node is scaled based on its degree, to highlight the most important nodes in the network. For Figures 2-5, color coding is used to show which section students belong to. Finally, for these particular graphs the placement of individual students is uniform (e.g., the third student in the second column is the same student in all four graphs).

Figure 2 shows the network of in-class connections. This corresponds to when students worked together either in lecture or recitation sessions. As one would expect, students were most likely to work with students from their same recitation sections. Nevertheless, one can also see that there are numerous connections across colors, which indicate that students worked with students from other recitation sessions as well. These collaborations would correspond to work that took place in the large-lecture sessions.

Figure 3 is the network of friendships. As one would expect, this graph was closely related to the in-class network. This suggests that students generally had positive relationships with the students they worked with in class, enough to consider them friends. The students belonging to the Seminar and Workshop sections had more connections than their peers in the Standard sections, as one would expect. These results are explored in depth in the following section.

Figure 4 shows the network of out-of-class connections. In many ways, this graph looks similar to the friendship network, but there are fewer relationships. This follows the pattern one might expect: students are most likely to work together in class, which might result in the formation of friendships, and in the strongest cases, students may also work together outside of class. While it is possible that some students may work out of class with students they do not identify with as friends, this seems less likely. Given that the out of class network is mostly a subset of friendship network, it seems that social connection may be necessary but insufficient to support academic collaborations.

Figure 5 is the network of contributors. This graph is directed, to indicate when students nominated other students as contributors to the class. In this graph, three students stand out as being nominated by many more students than others. Two of the three identified students belonged to the Standard B section, and one was in the Seminar section. All three of these students identified as White, and two of three identified as male. This pattern aligns with what Stinson (2008) has called the "White male math myth," that White males inherently possess a greater mathematical ability than other people. With the given dataset, it is not possible to determine whether or not these students actually contributed more than other students, simply that they were recognized as contributors by the most students.

These students were nominated by students from all sections, not just their own subsections. This means that despite the instructor's efforts to promote equitable participation, there were a few students who were clearly identified as high status in the class. Recall that to nominate a student in the sociometric survey, one needs to know the student by name. Beyond these three highest status students, there were a number of students in the Workshop section that identified one another as valuable contributors. This is another indicator of the cohesiveness of the group in the Workshop section. There were also a number of students who identified themselves as valuable contributors, as shown by the loops in the graph.

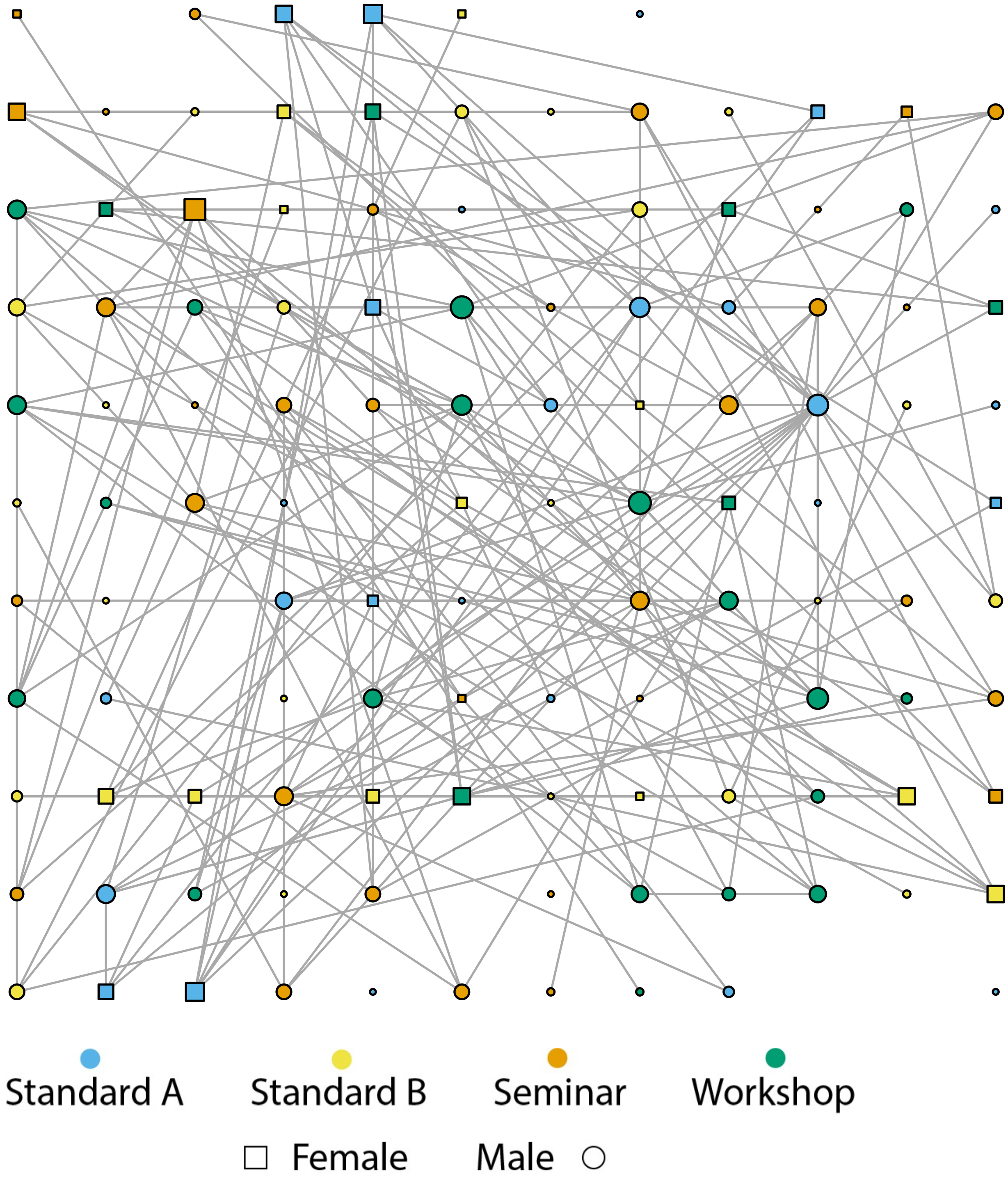

Figure 2. In-class network (final).

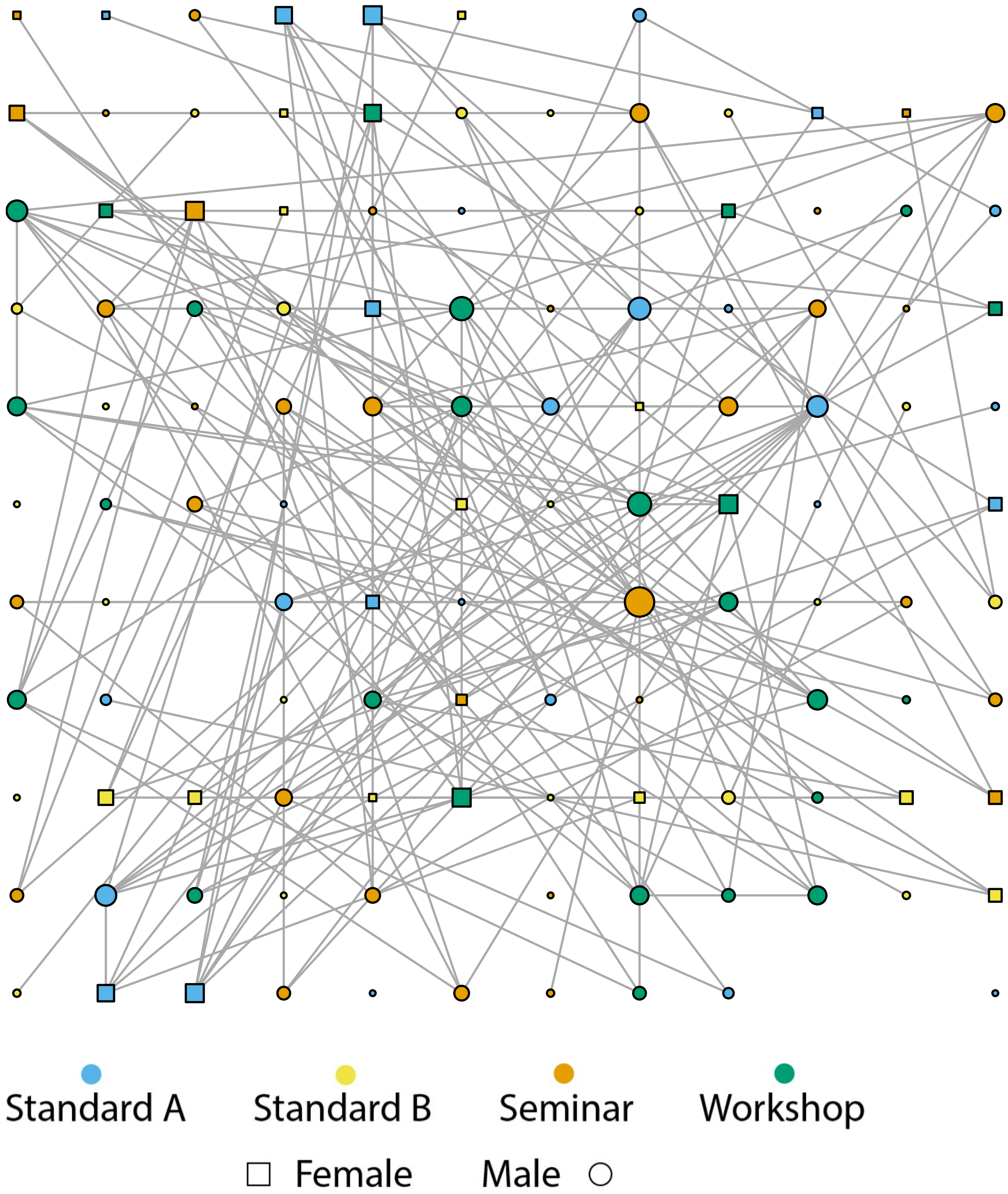

Figure 3. Friendship network (final).

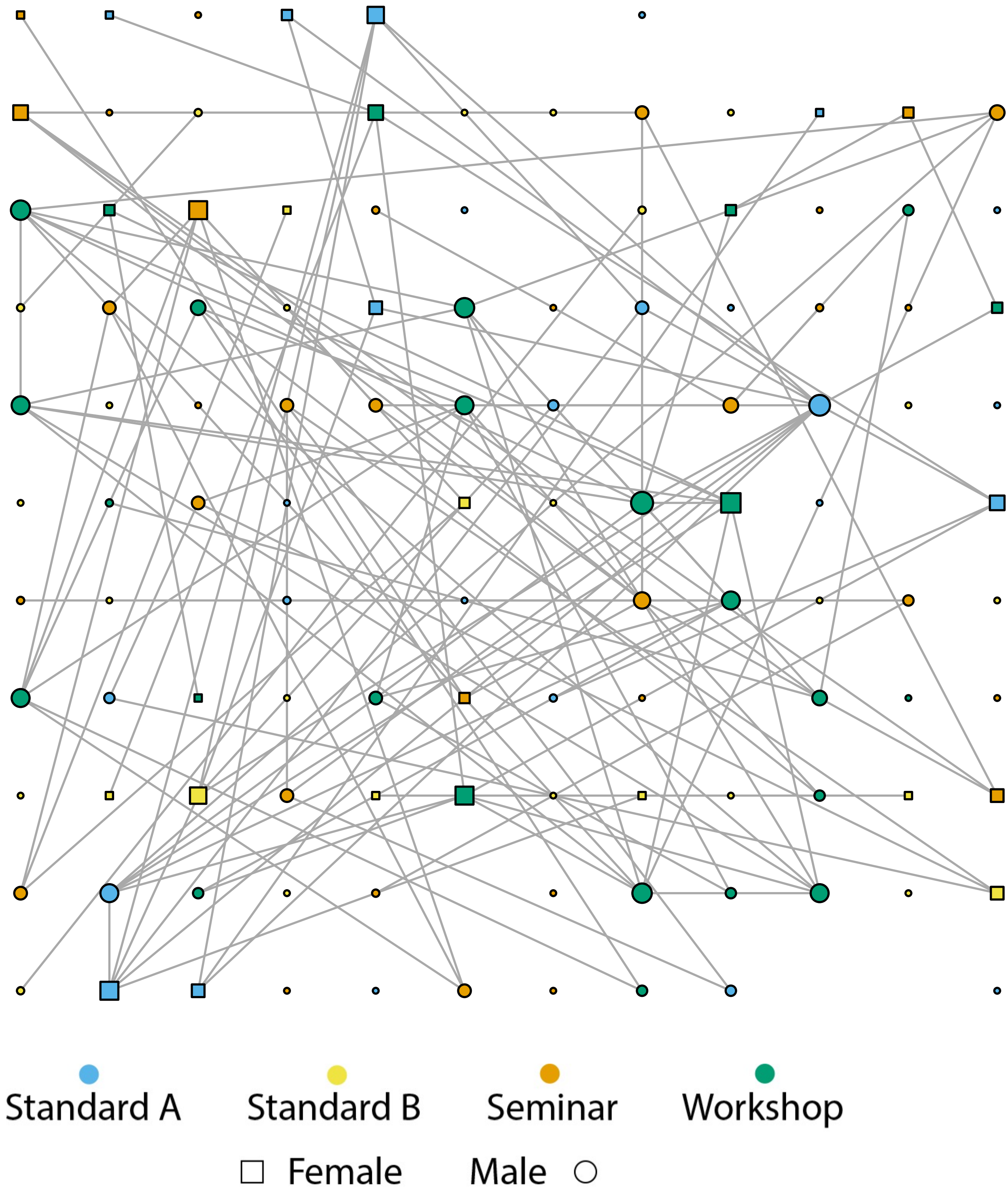

Figure 4. Out-of-class network (final).

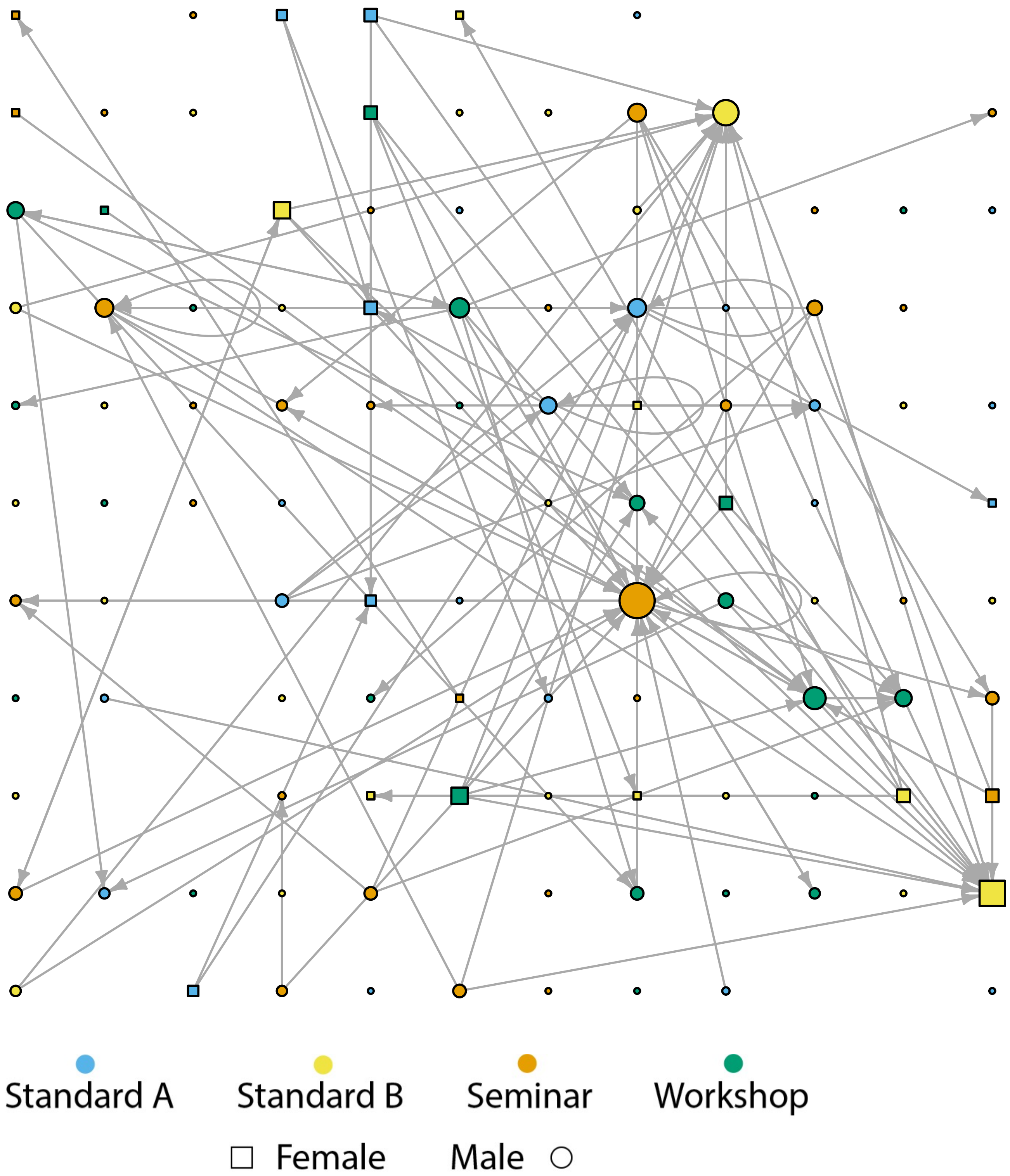

Figure 5. Contributors network (final).

Tables 10 and 11 show the average degree as broken down by gender (Table 10) and race (Table 11). Table 10 indicates that the average degree for male students was higher than for female students, for in-class, friends, and contributors; the average degree was the same for out-of-class networks. Table 11 shows the average degree of students by race. The results for out-of-class networks show that African American, Asian, and Hispanic students were the most connected. The result for Asian students mirrors what was found with the MWP, and the results for African American and Hispanic students are likely a result of the co-calculus sections that many of them were enrolled in. As a whole, the results in Tables 10 and 11 should be interpreted with care, because both race and gender are correlated with whether or not students were in a co-calculus experience. Finally, the highest degrees for being recognized as contributors were for Asian and White students, which is consistent with racial narratives in the US (Nasir & Shah, 2011).

Table 10. Average degree by gender (final).

	Female	Male
In-class	2.37	3.01
Friends	2.33	3.14
Out-of-class	1.91	1.91
Contributors	1.28	1.74

Table 11. Average degree by race (final).

	African American	Asian / Pacific Islander	Hispanic	International	Multiple Ethnicities	White	Unknown
In-class	3.25	3.66	3.25	2.00	2.45	2.10	2.33
Friends	3.75	4.05	2.88	2.00	2.91	2.39	2.23
Out-of-class	3.25	3.23	2.28	1.00	1.81	1.30	0.81
Contributors	1.25	1.86	1.10	0.00	1.72	2.45	0.08

Networks by Section

This section focuses on individual sections. Table 12 provides a summary of relevant network properties for each of the subnetworks. On nearly all measures of cohesion the Workshop section outperforms the other sections. Additionally, one can see the same patterns as above in terms of the nesting of the networks: in-class and friendship networks had a similar edge density, while the density of the workshop sections was lower.

Table 12. Properties of the subnetworks (end of semester).

		Standard A	Standard B	Seminar	Workshop
Edge Density	In-class	0.046	0.028	0.055	0.132
	Friends	0.054	0.017	0.066	0.145
	Out-of-class	0.030	0.005	0.036	0.132
	Contributors	0.014	0.007	0.018	0.025
Centralization	In-class	0.147	0.086	0.127	0.188
	Friends	0.171	0.068	0.298	0.215
	Out-of-class	0.163	0.024	0.116	0.188
	Contributors	0.085	0.081	0.017	0.099
Transitivity	In-class	0.333	-	0.429	0.506
	Friends	0.341	0.000	0.267	0.538
	Out-of-class	0.329	0.231	0.352	0.517
	Contributors	0.158	0.000	0.158	0.273

Table 13 shows the average degree by section and gender. The table indicates that the average degree for students in the Workshop section was higher than all of the other sections. The results were particularly profound for the male students in the sections, who made up the majority of the students. The Seminar also had some impact on the social networks, which were in general more cohesive than in the Standard sections.

Table 13. Average degree (final).

F / M	Standard A	Standard B	Seminar	Workshop
In-class	2.29 / 2.29	1.93 / 1.68	2.86 / 3.08	3.00 / 4.89
Friends	2.50 / 2.82	1.47 / 1.00	2.42 / 3.42	3.71 / 5.16
Out-of-class	1.79 / 1.47	1.00 / 0.21	2.57 / 1.65	3.42 / 4.32
Contributors	0.92 / 1.23	1.60 / 0.89	0.85 / 2.38	1.71 / 2.16

Table 14 disaggregates the out-of-class networks by section and race. While friendship may result from the co-calculus sections, whether or not students work together outside of class is likely the strongest measure of their relationships. The results in Table 14 are profound. The average degree in the Workshop section dwarfs all of the other sections, and the results are most profound for African American and Hispanic students, for whom building greater social cohesion is often a target of co-calculus programs. The results should be interpreted with some caution, as many cells in the table have very few students. Still, the overall degree (final row) shows that the Workshop section is much higher than all other sections.

Table 14. Average degree (out-of-class; final).

	Standard A	Standard B	Seminar	Workshop
African American	0.00	-	-	6.50
Asian / Pacific Islander	5.00	0.00	2.50	5.43
Hispanic	0.67	0.89	2.00	3.92
International	0	-	-	2.00
Multiple Ethnicities	2.17	-	1.00	2.00
White	1.11	0.75	2.00	-
Unknown	1.43	0.25	0.75	1.00
Overall	**1.61**	**0.55**	**1.85**	**4.08**

Given that the goal of the co-calculus programs was to improve social cohesion, the following analysis focuses only on friendship networks and out-of-class collaborations, given issues of space. See Figures 6 and 7. The differences between sections are stark. While there was some friendship in both of the Standard sections, this did not necessarily translate into out-of-class collaborations. In Standard B there was very little out-of-class collaboration. In Standard A there was some out-of-class collaboration, but it was mostly limited to White and Asian students. While the Seminar did have a reasonable amount of out-of-class collaboration, it was much less than in the Workshop. Not only did the Workshop section have the strongest friendships, this translated to how students were working with one another outside of class. This showed an integration between academic and social relationships. Participation in these various networks is quantified in Table 15.

Table 15. Percentage of students in the network (final).

	Standard A	Standard B	Seminar	Workshop
Friendship	51.6%	47.0%	75.8%	92.3%
Out-of-class	38.7%	17.6%	57.6%	96.2%

Given that friendships (social) and out-of-class collaborations (academic) are theoretically most salient, they were further explored. Figures 8 and 9 show how the social networks for friends and out of class contributions changed over time for the Workshop section and Seminar sections. Rather than representing gender on the network graphs, these figures show which co-calculus session (if any) students participated in.

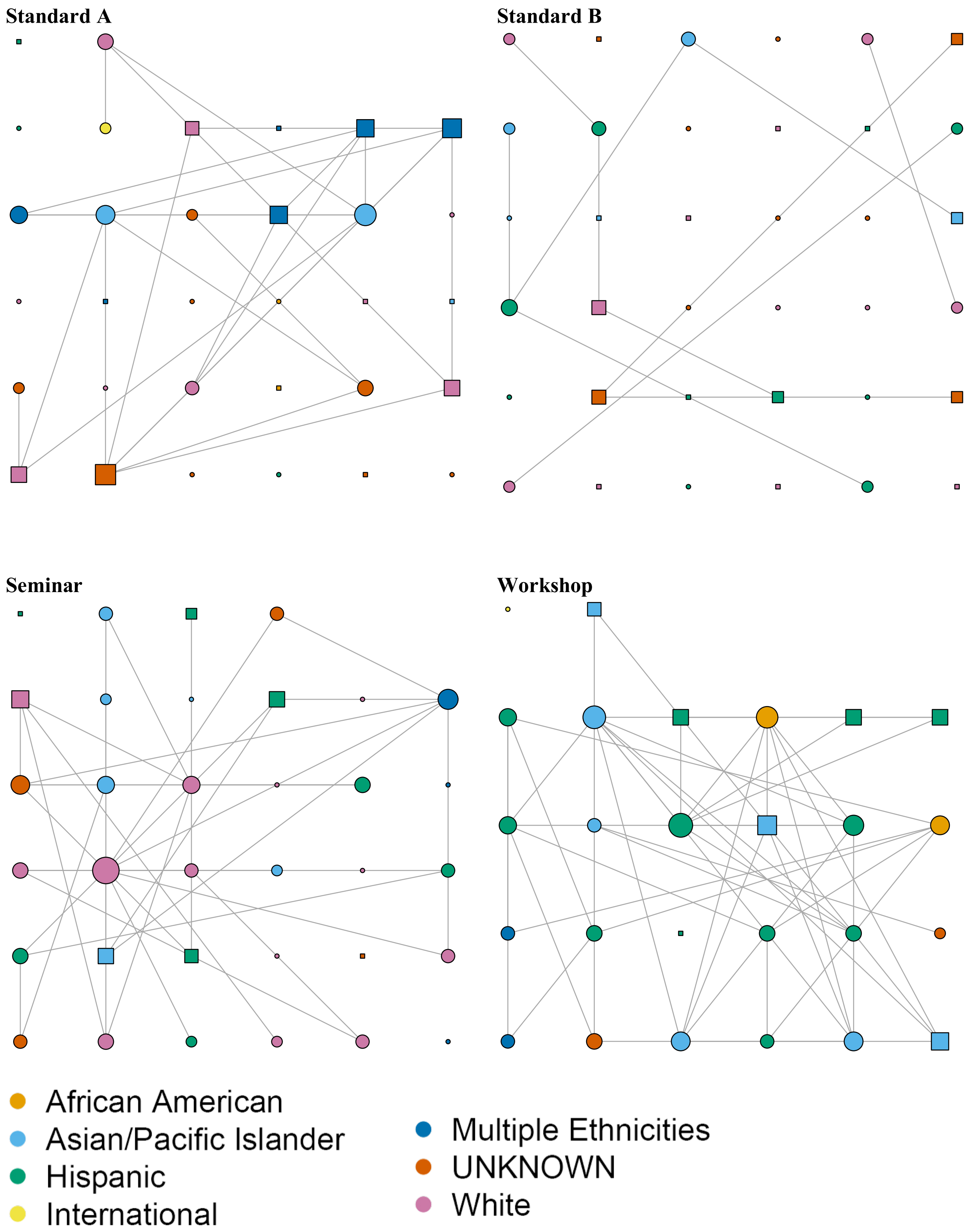

Figure 6. Friendship networks (final)

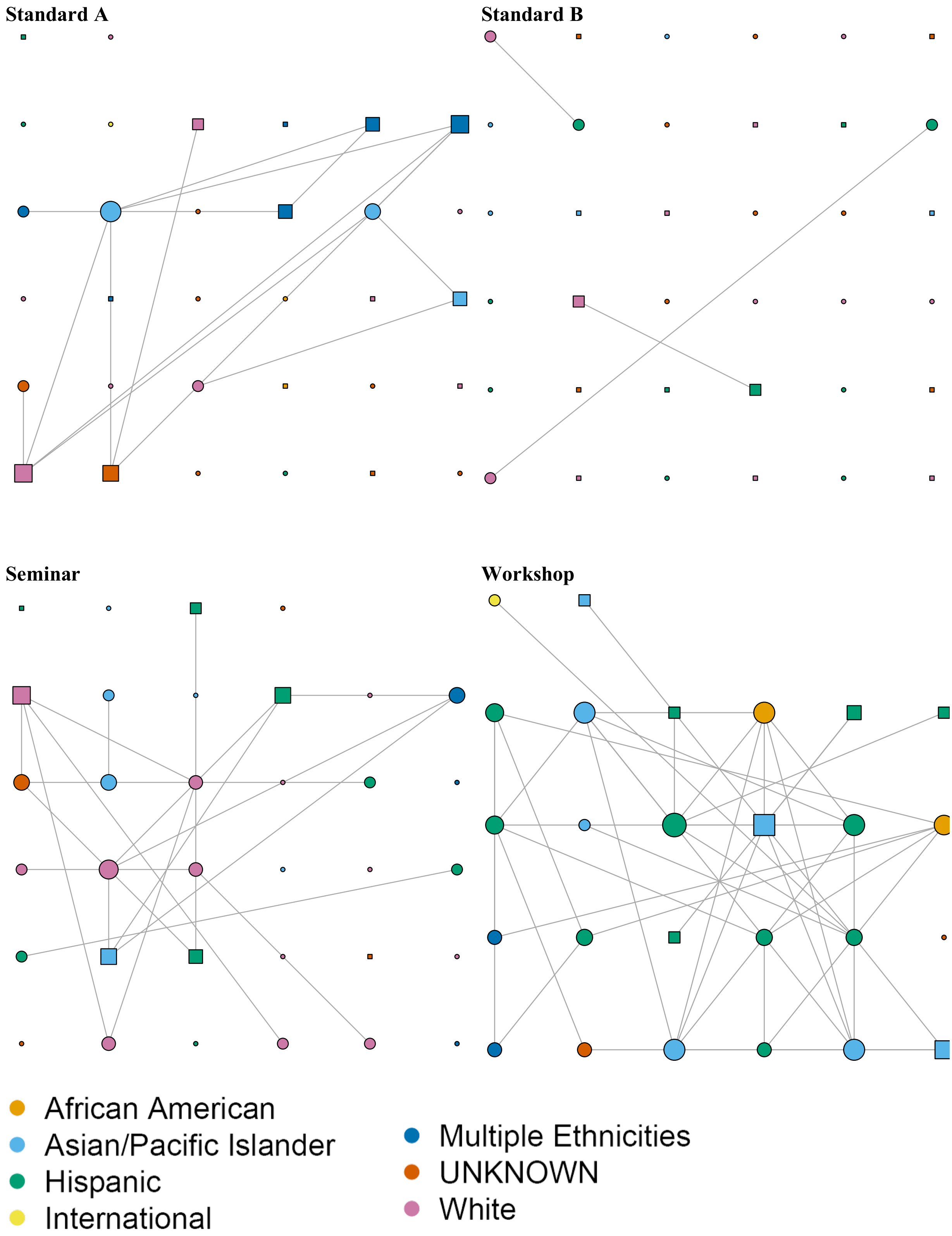

Figure 7. Out-of-class networks (final).

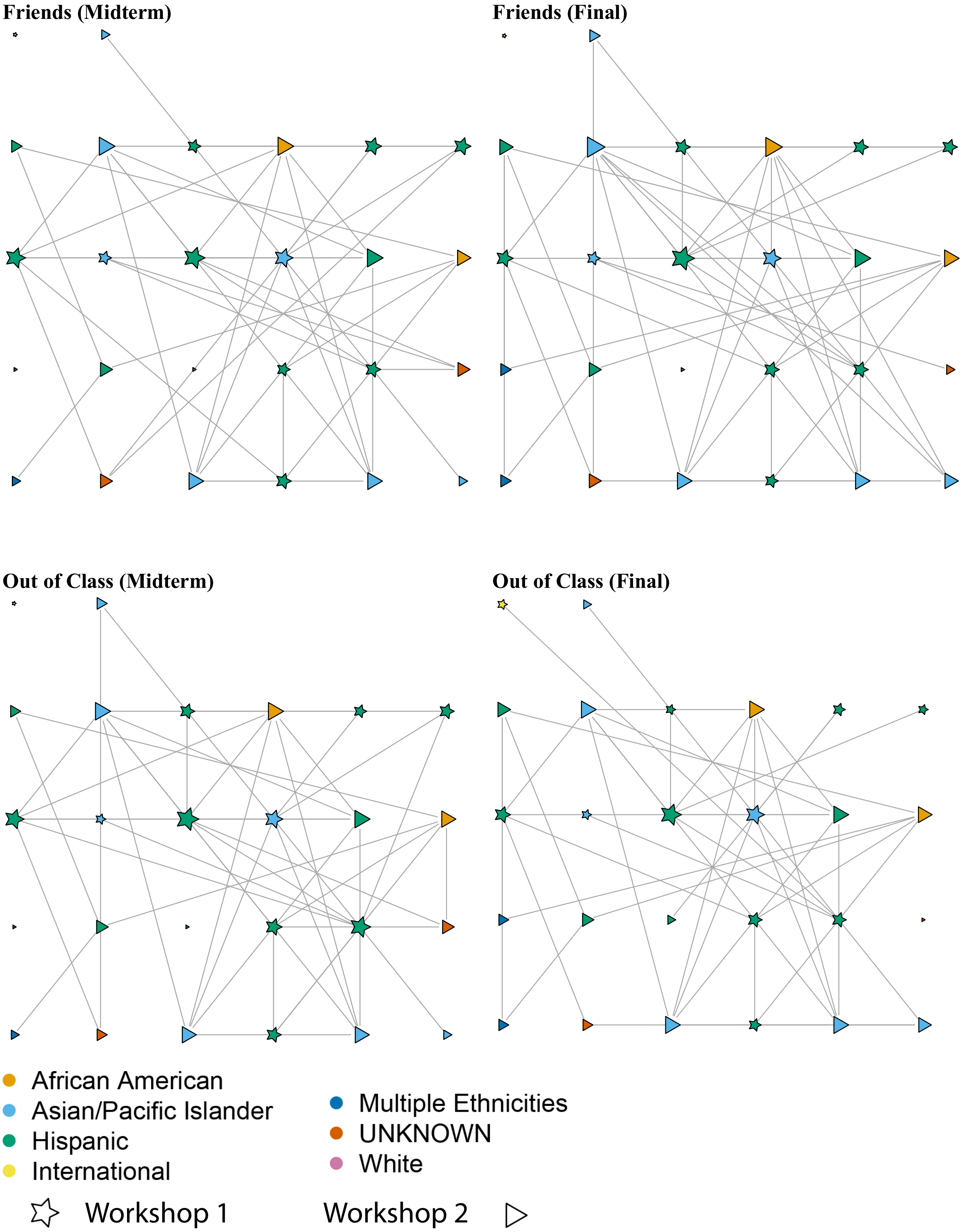

Figure 8. Midterm vs. final networks (Workshop).

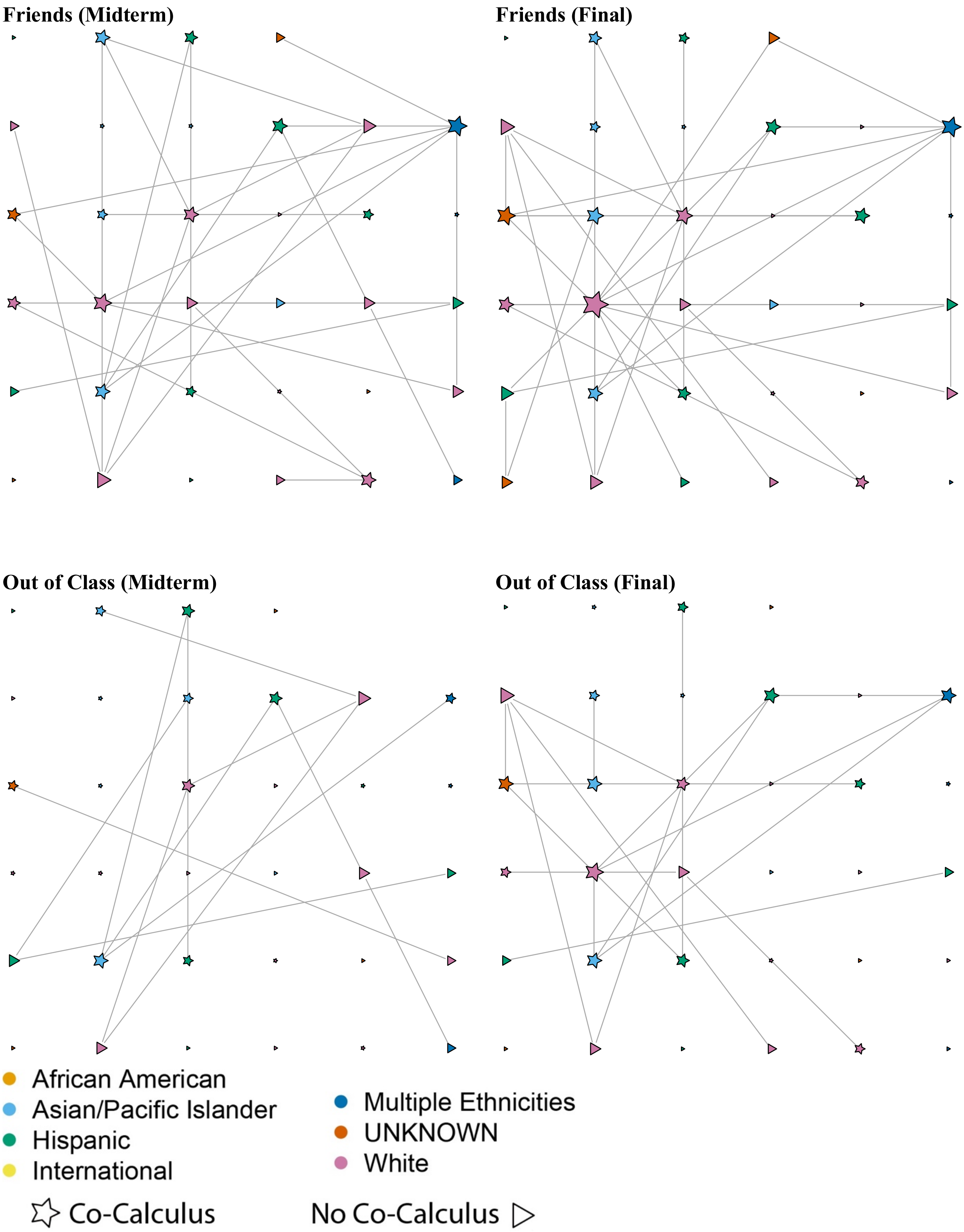

Figure 9. Midterm vs. final networks (Seminar).

Table 16 provides a quantification of the network graphics, in terms of percentage of students who were a part of the network. Both of the Workshop sub-sections built robust friendship and out-of-class networks early in the semester, and these persisted to the final. Even more interesting, the graphs show that a number students in Workshop 1 (stars) were connected to students in Workshop 2 (triangles). This indicates that students built strong relationships across their entire recitation section, not just their workshop subsection.

Table 16 also shows a contrast between the Seminar and Workshop, where the Workshop built stronger networks both for friends and out-of-class collaboration; this likely contributes to the greater success of Workshop students. Nevertheless, it is noteworthy that students in the co-calculus Seminar did build rather strong out-of-class collaborations, especially seeing that as commuter students they are generally more socially isolated than others. Unlike the Workshop, the out-of-class network for commuter students continued to evolve considerably between the midterm and final.

Table 16. Percentage of students in the network (by co-calculus).

	Seminar (Co-Calculus)	Seminar (No Co-Calculus)	Workshop 1	Workshop 2
Friendship (Mid)	70.6%	62.5%	90.1%	86.7%
Friendship (Final)	82.4%	68.8%	90.1%	93.3%
Out-of-Class (Mid)	52.9%	43.8%	90.1%	86.7%
Out-of-Class (Final)	76.5%	37.5%	100%	93.3%

Networks and Performance

To understand the relationship between various social networks and final exam scores, a set of correlations were run (see Table 17). This table shows that the various social characteristics (i.e. in-class, friendship, and out-of-class) were highly correlated, and that they were much less correlated with nomination as a valuable contributor. The correlation between all network features and final exams was small but positive.

Table 17. Correlation between degree characteristics and final exam.

	Friends	Out of Class	Contributors	Final Exam
In Class	0.86	0.78	0.45	0.20
Friends	-	0.82	0.61	0.21
Out of Class	-	-	0.39	0.20
Contributors	-	-	-	0.17

Multiple regression was used to test if out-of-class degree (end of semester) significantly predicted participants' final grades. The model was significant ($R^2 = 0.038$, $F(1, 122) = 4.823$, $p < 0.05$), and out-of-class degree was a significant predictor ($\beta = 1.93$, $p < 0.05$). As one would expect given the high correlation between in-class, friends, and out-of-class degree, these three variables were all significant predictors in their own individual models (results not reported here). Yet, when taken together, the model was not significant ($R^2 = 0.046$, $F(3, 120) = 1.942$, $p = 0.126$), with the following results: in-class degree ($\beta = 0.86$, $p = 0.61$), friends degree ($\beta = 0.60$, $p = 0.70$), and out-of-class degree ($\beta = 0.60$, $p = 0.70$). This indicates that looking at all three variables together added little new information to the model. Interestingly, taken by itself, degree in the contributors network ($\beta = 1.28$, $p = 0.055$) was not a significant predictor ($R^2 = 0.029$, $F(1, 122) = 3.753$, $p = 0.055$). This indicates that social integration (e.g., in various friendship or collaboration networks) was more important than actually being identified by peers as a valuable contributor in the class.

Conclusion

This paper compares three efforts to improve student success in introductory calculus: (1) active learning labs, targeted at all students, (2) a small-group seminar for commuter students, and (3) a workshop model targeted at "underserved" students in STEM. Each of these efforts was organized around a similar aim: to provide a collaborative learning experience that helps students integrate the academic and social dimensions of their lives. Of the three efforts, the workshop model appeared most successful. Workshop students had significantly higher passage rates, and more cohesive social networks.

At a base level, it appears that the department's efforts to support greater student community through twice-weekly recitations and a common student workspace for office hours did not foster a high-level of social cohesion. While it is possible that these features increased student success compared to historical iterations of the course, those analyses were not a part of this paper. Nevertheless, it is clear that the Standard sections did little to build strong networks between students, especially in the Standard B section. Moreover, the networks that did form in the Standard A section were organized around typical racial patterns (mostly White and Asian students), which has important equity implications. The situation was different in the Seminar section, where friendship networks did form, and the networks seemed to transfer somewhat to out-of-class collaborations.

While this provides some evidence of the integration of academic and social aspects of college life, this partial integration was not enough to impact student outcomes, taken from the perspective that students in all sections had similar academic backgrounds. Yet, a positive alternative interpretation is that commuter students are naturally "at-risk," and they may have actually done worse than their peers had it not been for this extra co-calculus support.

The results for the Workshop section were impressive. Student outcomes were greatly enhanced, and there was evidence of strongly integrated academic and social networks; nearly all of the students in the Workshop section reported strong connections to their peers, both academically (working together out of class) and socially (identifying each other as friends). While the connection between outcomes and social networks is only correlational, not causational, this provides further evidence in favor of the theoretical underpinnings of the Workshop program. Because students had high expectations and high support, they were pushed to do their best in a supportive community, rather than being forced to undergo remediation, which would be counter to building stronger math identities. The results in the Workshop section are particularly impressive considering that the Workshop session featured a higher proportion of African American and Latinx students that the other sections, who are less likely than their White counterparts to be integrated socially than their White peers.

Looking at statistical models, all of the collaboration networks were significant predictors of student success (when taken individually). In contrast, being nominated as a valuable contributor was not significantly related to final exam score. This suggests that social integration is more important than being seen as smart. It also suggests that how students identify "smartness" may be more related to race and gender than actual understanding or performance in mathematics. While these results are only suggestive, they do align with prior research (Stinson, 2008) and do have important equity implications.

Recommendations

Why is it that the Workshop section had significantly greater success than the other sections? The results suggest that Workshop students were better able to integrate the academic and the social, but why was that the case? Following prior work, it is likely that the ability to identify with a positive honors program was of value. The additional time on task (as compared to other sections) most likely helped too. This suggests that others can benefit more from framing their co-calculus programs rather than as remediation.

Ultimately, more research is required. This manuscript does not interrogate the lived experiences of students within the Workshop program, nor does it follow them over time. Given the promise of the Workshop program, these are both areas for future research. In either case, mathematics educators should seriously consider the nature of their community-building efforts, particularly through co-calculus, as there is no guarantee that they will actually build community as desired.

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