Designing Instructional Supports for Mathematical Explanations

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This paper reports on a design-based research study focused on supporting students to generate and reflect on explanations of mathematics. Analyses suggested that effective guidance involves: (1) helping students overcome the perceived need for expertise to construct and critique explanations (the "expert barrier"), and (2) providing explicit prompts for self-assessment and reflection. These principles resulted in the design of a standardized assessment tool to support students to engage in peer- and self-assessment, the preliminary analysis of which is included.

Introduction

Explanation and reflection play crucial roles in learning mathematics (e.g., NCTM, 2000). More than just communicating understanding, explanation plays an important role in developing understanding (Chi et al., 1989; Lombrozo, 2006) and self-regulation (Chi et al., 1994). Accordingly, there is widespread agreement about the importance of explanation, both in mathematics education (CCSSI, 2010; NCTM 2000) and science education (AAAS, 1993; NRC, 1996). Yet, explanation remains ill-defined, both for researchers and students (Braaten & Windschitl, 2011), and unfortunately, students often fail to learn to explain mathematics in deep and meaningful ways as a result of traditional instruction (e.g., Aud et al., 2011; Schoenfeld, 1988). Thus, the present study focused on addressing the research questions: *(a) what are the barriers for students constructing and reflecting on explanations?; and, (b) how does one design a learning environment to overcome these barriers?*

Peer-assessment activities afford opportunities for students to develop a sense of "high quality" explanations by constructing, critiquing, and reflecting on explanations (cf. Ford & Forman, 2006; Sadler, 1989). In this way, peer-assessment acts as a precursor to self-assessment (Black, Harrison, & Lee, 2003), as developing metacognitive processes at the social level supports students' internalization of such processes (cf. Palinscar & Brown, 1984; Schoenfeld, 1985; White, Frederiksen, & Collins, 2009). However, most theories of peer- and self-assessment do not account for the specifics of learning *mathematics* (cf. Topping, 2009). As a design-based research study, this paper contributes refinements to local theories of explanation and assessment specific to the domain of mathematics (cf. Cobb et al., 2003; Gravemeijer, 1994). Furthermore, I report on the preliminary results of using a peer- and self-assessment tool designed on these principles (to be included in the final paper – omitted now due to space).

Method

Participants were community college students ($N = 20$) in a remedial algebra class in an urban school in the San Francisco Bay Area. As an instructor and researcher I collected copies of students' written work, in-class reflections, and video recorded each class session using a stationary camera. For comparison, I observed and recorded 5 class sessions and obtained copies of student work from a parallel section ($N = 34$) of the same course taught by another instructor without the experimental intervention.

The intervention consisted of a number of mutually-supportive activities for promoting explanation and reflection on explanations, including “traffic lights” (explained below; cf. Black et al., 2003), daily reflections, assessment problems of real and hypothetical student work, and use of the standardized assessment tool. Because peer- and self-assessment were novel activities for students, significant instructional time was dedicated to providing scaffolding and support (cf. Topping, 2009).

Results and Analysis

Through analysis of classroom videos and student written work I devised two design principles and created an assessment tool embodying them. In what follows, I describe some of the challenges uncovered through implementing the intervention and how the assessment tool seems to overcome them.

Design principle 1: address the “expert barrier”

Most students believed that only “experts” are capable of explaining mathematics, which acted as a barrier to their engagement with explanation activities. This was evident in classroom videos, in which students were reticent to engage in explanation, instead asking the instructor to “just tell them” the answer. In this sense, explanation refers to understanding *why* the mathematics works, not just focusing on *how to do it.* In their reflections, students frequently made statements such as “why do you always ask us why?” or “you are asking us to explain too much.” Evidently, the expectations of this course were different from students’ prior conceptions of what it means to understand mathematics. Nevertheless, it was clear that students understood the expectations of this particular course. During a discussion one day in the middle of the semester, when a number of students were asking for the answer, another student, Sigfried, exclaimed “you know he’s not going to tell us the answer!” On another day, Teresa noted before class, “I’m not sure exactly why we have to explain, but I’ll go with it. I guess it kinda helps when you go back trying to figure out what you did.”

Such statements and interactions provided evidence that students perceived explanation primarily as a means of *communicating* understanding, not *constructing* it. For instance, Teresa reflected “I can’t explain what I learned because I’m confused. I understand rise/run but the rest is confusing.” In this way, the perception that only experts can explain mathematics acted as a barrier preventing Teresa from engaging in explanation, even though attempting to explain slope would help her to resolve her confusion.

Above I describe an “expert barrier” – the perceived need for expertise prevents students from developing expertise. Yet, given that explanation is a constructive activity, it is one of the very activities that students needed to engage with in order to *develop* expertise. Explanation provides students with an opportunity to surface, examine, and reflect upon their current understandings, refining them toward more normative understandings (Chi et al., 1994). To help students overcome this barrier, one must address the low mathematics self-efficacy which is prevalent in remedial mathematics (cf. Bandura, 1997). Moreover, one must help students develop a sense of internal mathematical authority, so that they can see explanation as a means for constructing meaning (cf. “connected knowing;” Boaler & Greeno, 2000).

Productive reflection requires students to ask specific questions to clarify or extend an explanation as a means of deepening understanding (in contrast to *general* statements, such as “I don't get it"). However, if students perceive that asking questions portrays a lack of understanding or expertise, they will be reluctant to ask them. Accordingly, the assessment tool attempts to overcome such perceptions by reframing the task of asking questions.

By focusing on *understanding* explanations rather than *assessing* them, students were better able to engage with peer-assessment tasks, and ultimately *assessed* the explanations regardless of their initial confusion. For an illustration, consider an excerpt from a lesson in which students used the standardized assessment tool to facilitate reflection on sample student work[1]:

> Instructor: *That's good, so what questions might you ask him?*
> …
> Dahlia: *I don't get it, because he's not explaining anything.*
> Julius: *I want him to say more about two people, I didn't really get that part.*
> Instructor: *Okay, so you want him to clarify what he meant about two people.*
> Julius: *Yeah, I don't really understand that part. When you have two people it's easier than to do it by yourself; what do you mean by that?*

Even though the task was framed as *understanding* the given explanations, it ultimately led to the *assessment* of the explanations, as evident in Dahlia's comment that he "wasn't explaining anything." In his reflection that day, Julius noted "I learned that explaining a problem *wrong* even with the right answer can be confusing." This shift in framing was effective because the perceived need for expertise was reduced. In sum, a successful intervention should both work to change students' perceptions of explanation (as a constructive activity) *and* provide entry points into explanation and reflection that don't require students to feel as though they are experts.

Design principle 2: provide explicit prompts for self-assessment

Students used "traffic lights" to self-assess how well they understood each lesson and homework problem (for 12 weeks of the course; cf. Black et al., 2003). Green represents a strong understanding, yellow represents a partial understanding, and red represents a lack of understanding. Students also gave justifications for the colors they chose. The purpose of traffic lights was made explicit to students, and students' self-assessments were frequently used as the basis of in-class activities and discussions.

Unfortunately, the accuracy of self-assessments showed little improvement throughout the semester. While students could easily identify "red lights," students frequently marked work that was mostly incorrect (e.g., answers that were off by many orders of magnitude) as green, overestimating their understanding (cf. Dunlosky & Lipko, 2007). Student reflections about traffic lights illuminated the source of some of their difficulties. For example, Vanessa stated:

> *"Red flag" doesn't help me - it only makes me frustrated I don't have an understanding of solving or graphing. That is my frustration for the entire homework.*

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[1] The work was a part of the Shell Centre's formative assessment lesson, "Golden Rectangles," which is freely available online at http://map.mathshell.org/materials/download.php?_leid=1226

Vanessa notes that a "red flag" doesn't help, because it provides insufficient *grist* for working through partial understandings. A general, non-specific method of self-assessment (such as the traffic lights) rests on the tacit assumption that students have the skills to self-assess and reflect, but not the predilection to do so. However, most students made vague, general comments such as "I'm confused" or "I don't get it," which are not very effective for self-assessment because they do not specify "what" exactly needs to be resolved. Based on these insights, the assessment tool provides specific scaffolding questions and exercises for self-assessment. For evidence of the tool's efficacy, consider a spontaneous comment made by Telma after class one day:

> *This new assessment form is a lot better than the traffic lights. It actually gives you questions to answer, which makes it a lot easier to use.*

In order to self-assess their problem solving, students need to identify warrants for their mathematical claims, answering the crucial question "how do you know?" about their work. This first requires that students perceive themselves as capable of assessing the validity of mathematical claims, thereby developing a sense of internal mathematical authority (cf. Boaler & Greeno, 2000). Accordingly, a successful intervention must not only teach students *how* to self-assess, but first help them come to understand that they *can* self-assess.

Design product: standardized peer- and self-assessment tool

The above principles were embodied into a standard form for peer- and self-assessment, designed to facilitate the internalization of social practice (Reinholz, 2013). The form asks students to assess work in a way that doesn't appear to require expertise (at least at the surface level), which appears to help them overcome the expert barrier. As students engage in peer-assessment, they are afforded opportunities to practice constructing explanations and receive feedback on their explanations, guided by questions and criteria that help them develop an understanding of what a "good" explanation looks like.

The assessment form consists of three prompts and a number of scaffolding questions; students fill out a single form in response to the solution to a single mathematics problem. With regards to the solution of a problem, the prompts ask students to: (a) describe the big picture (ie. the approach to solving the problem), (b) assess accuracy (justifying how they know), and (c) ask questions to clarify or extend the mathematics in the problem. The scaffolding questions complement the prompts by providing explicit guidance on how to perform the reflection. For instance, with respect to assessing accuracy, sample questions include "were any assumptions made justified?" or "was the problem solved in multiple ways?" which are tools students can use to assess the response's accuracy. In this way, the assessment tool provides students with means for developing internal mathematical authority. Beyond peer- and self-assessment, the scaffolding questions are designed to be used to guide classroom presentations and provide a basis for productive mathematical discussions.

Although in-depth analysis is beyond the scope of the present paper, evidence of the efficacy of the assessment tool was given above, both in how it supported students' engagement with the Golden Rectangles problem and in Telma's comment about how it was more effective traffic lights. In the next iteration of the design-based research study I will study the efficacy of the assessment tool in-depth.

Discussion and Conclusion

To support students' explanations and reflections in mathematics, an effective design should: (1) address the perceived need for expertise to construct and critique explanations (the "expert barrier"), and (2) provide explicit prompts for self-assessment and reflection. These principles were embodied into an assessment tool, a preliminary analysis of which was provided.

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