TRANSITIONING FROM EXECUTING PROCEDURES TO ROBUST UNDERSTANDING OF ALGEBRA

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Algebra instruction should help students navigate the transition from executing procedures to a more robust understanding of algebra. There is little empirical evidence, however, linking promising instructional practices to such understandings. The Algebra Teaching Study seeks to develop tools for measuring connections between algebra teaching and students' learning outcomes. To demonstrate the use of these tools, we present a comparison of two algebra classrooms, where instructional differences were related to differences in student outcomes.

Keywords: Algebra and Algebraic Thinking; Instructional Activities and Practices; Middle School Education; Problem Solving

Introduction

While growing attention has been given to algebra teaching and learning, there is still a lack of empirical evidence linking teachers' instruction and students' understanding of algebra. The purpose of the Algebra Teaching Study is to develop measurement tools to determine which classroom practices help students navigate the transition from executing procedures to the development of robust understanding of algebra. Our classroom observation scheme, Teaching for Robust Understanding in Mathematics (TRU Math), is being developed using classroom observations in Michigan and California. To measure students' gains in robust understanding, we articulated a set of dimensions defining robust understanding of algebra, which we call Robustness Criteria (RC). We adapted and constructed tasks to measure student understanding along these dimensions. We illustrate these tools by presenting comparative cases in which differences in instructional practices of two classes related to differences in gains in algebra achievement. For this paper, we focused on algebraic representations, due to their central role in algebra. We first present our framework for measuring robust understanding of algebra.

Theoretical Framework

The study focused on student understanding of algebra word problems. This is due to their central role in the algebra curriculum, as well as students' struggles with them. Focusing on word problems enabled us to examine a range of student skills related to robust understanding, including: sense making, modeling, representational and procedural skills. In this section, we will elaborate our definition of robust understanding.

Robustness Criteria Framework

The robustness criteria were developed in consultation with a large body of literature, including: the *Principles and Standards for School Mathematics* (NCTM, 2000), the Common Core State Standards for Mathematics (2010), studies of how students solve problems (e.g., Schoenfeld, 2004), and algebraic habits of mind (Driscoll, 1999). The robustness criteria (RCs) serve two major purposes in our project: guiding task selection and analysis, and focusing our attention during classroom observations, for example, on teacher actions that might promote fluency with algebraic representations. Below, we list our five criteria. We elaborate RC3 because it is the focus of this paper.

RC #1: Navigate language to make sense of the problem situation.

RC #2: Identify relevant quantities and relationships between them.

RC #3: Represent quantitative relationships. Relating and representing multiple varying quantities is a core feature of a functions-based approach to algebra (e.g., Chazan, 2000). Algebraic representations include coordinate graphs, bivariate tables, diagram or pictures, and variable equations (or systems of equations). Further, the Understanding Patterns, Relations, and Functions Standard states that students should be able to "represent, analyze, and generalize a variety of patterns and tables, graphs, words, and when possible, symbolic rules" (NCTM, 2000, p. 222). Schoenfeld (2004) identified building a situation model and building a diagram or other appropriate representations as being important aspects of student knowledge for solving word problems. The use of multiple representations facilitates students' development of mathematical concepts (e.g., Stein et al., 2009) and their efforts to carry out problem solving tasks (e.g., Greeno & Hall, 1997). Tasks with high cognitive demand should have the potential to be represented in multiple ways (Stein et al., 2009). Therefore, students should be given the opportunity to use multiple representations in problem solving activities.

RC #4: Executing algebraic procedures and checking solutions.

RC #5: Explain and justify reasoning.

These criteria represent the proficiencies required to solve rich, contextual algebraic word problems (Wernet et al., 2011), which we henceforth refer to as contextual algebraic tasks.

Method

Over the course of two years, we have collected data from ten 8th-grade algebra classrooms. We administered pre-tests at the beginning of the school year to document students' initial proficiencies with contextual algebraic tasks. During the school year, we observed each classroom eight times, capturing lessons involving contextual algebraic tasks whenever possible. We then administered post-tests at the end of the school year to document students' growth in understanding as a result of a year of classroom instruction.

The pre- and post-test assessments were comprised of three, multi-part tasks drawn largely from the Mathematics Assessment Resource Service (MARS) assessments. Students' work received two types of scores: a holistic score based on rubrics that aligned with the MARS rubrics, and RC-specific scores, with points awarded related to each of the robustness criteria.

For the purposes of scoring, RC3 (interpreting quantitative relationships) was further elaborated to RC3a (generating representations) and RC3b (interpreting and making connections between representations). The RC 3a scores reflect, in part, students' spontaneous generation of a representation to complete a task in addition to the correctness of the representation generated. These scores were then compiled to provide overall class scores, allowing us to capture student growth, at the classroom level, in robust understanding of algebra across the school year.

Across the observed classrooms, teachers used a variety of curricula (four of the classrooms used an NSF-funded curriculum while the others used a traditional, district adopted text). Thus, the videos we have collected provide a range of lessons from those focusing on traditional story problems to those using more open-ended tasks. This collection of videos has been used both to identify teaching moves warranting attention in our observational tool, as well as to test the observational tool during the development process.

For the purposes of this paper we made a holistic characterization of the instructional practices around representations. We focused on five randomly selected students from each of two classrooms, A and B, in a comparative case study. Class A was comprised of low-tracked students in a suburban school in the Midwest and used an NSF-funded curriculum. Class B included de-tracked students and used a traditional text in a large urban setting on the West Coast. The case study addressed the following questions:

1. How does student use of representations in written algebraic tasks differ between two 8th grade algebra classrooms?
2. How do teachers' approaches to supporting students' use of representations differ between these two classrooms?
3. How does variation in student performance on tasks involving representations relate to the differences in instructional practices?

Illustrative Results

In our observations of Class A and Class B, we noticed significant differences in classroom practices. Students in Class A interacted with representations on a regular basis. Using a functions-based approach to learning algebra (cf. Chazan, 2000), tasks in the curriculum regularly asked students to generate equations, tables, and graphs, and to make connections between these representations. Additionally, Teacher A consistently pressed and supported students to make explicit their generation of representations and make connections between them through questioning, re-voicing, drawing attention to context and other visual cues, and building on previous knowledge.

In contrast, students in Class B spent much of their time taking notes on specific procedures, working on exercises and showing steps in their work. During independent or partner work time, students were primarily engaged in practicing procedures demonstrated in class lecture. Students had few opportunities to generate or interpret representations.

There were differences in students' learning growth both in terms of overall scores and in terms of students' use of representations, measured by RCs 3a (generating representations) and 3b (interpreting and making connections between representations). Table 1 shows the average total scores across the two classrooms on the pre- and post-assessments and the performance in understanding the use of the algebraic representations. Recall that students in Class A were placed in a low-tracked classroom and scored lower than Class B on the pre-assessment. By the end of the study, however, the performance gap had closed; Class A showed evidence of greater growth in RC3. Although students in Class B correctly generated and interpreted representations more frequently on the pre-assessment, the students in Class A finished the school year with higher scores on tasks involving representations.

Thus, initial evidence indicates that there are differences in students' growth in RC3 across classrooms, and that our assessments can capture differential growth.

Table 1: Scores across Two Classrooms

	RC3a		**RC3b**		**Total score**		**Difference**
	Pre	Post	Pre	Post	Pretest	Post-test	
Class A	0.6	4	3.6	6	20%	41%	21%
Class B	1.6	3	4.4	5	26%	32%	6%

The results shown illustrate the potential to document the relationship between students' learning with factors related to teachers' specific instructional practices.

Discussion

National discussions increasingly emphasize the importance of all students studying algebra by eighth or ninth grade. If the push for all students to study algebra is to lead to robust understanding, rather than only superficial skill with algorithms, mathematics educators must have a better understanding of which classroom processes will help build robust understanding.

Our Algebra Teaching Study has developed a system for classroom observation and task-based forms for student assessment that can be used for research linking differences among patterns of activity in classrooms to differences in student acquisition of robust understanding. The cross-case comparison presented here shows that our measures document such differences. Although a comparison across two classrooms is only suggestive of connections between teaching and learning, future work with larger numbers of classrooms can reveal whether such connections are typical. Development of these tools is a significant first step in an important line of research that is scant in the literature.

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