Capturing What Counts: Classroom Practices That Lead to Robust Understanding of Algebra

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Despite growing support for a student-centered, reform-oriented approach to mathematics instruction, there is still a lack of empirical evidence linking such an approach to improved student outcomes. Accordingly, the goals of the Algebra Teaching Study are to: (1) develop an observation scheme for capturing "effective" algebra classroom practices, (2) adapt measures to assess the robustness of students' understanding of algebra, and (3) begin to establish empirical support for connections between these measures. Here we focus on the development of the observation scheme. By attending to general aspects of mathematics instruction as well as aspects of instruction specific to what we have deemed Contextual Algebra Tasks (CATs), we hope to achieve the necessary resolution in our observations to draw inferences about specific classroom practices leading to greater student understanding.

Introduction

Despite the lack of a conceptualization of a "central core" of algebra (Mathematical Association of America, 2007), through decades of mathematics education research some major themes have emerged: (1) algebra as a generalization of arithmetic operations, patterns, and structures; (2) using multiple representations of functions, and (3) using algebra to model situations and solve problems (Kieran, 2006). Complementary to this conceptualization of algebraic understanding, there is a growing body of literature supporting a reform-oriented approach to mathematics instruction, which consists of a shift of classroom focus from teacher to student, with students engaged in rich, problem-solving activities, from which mathematics emerges (Gravemeijer, 2004; Schoenfeld, 1992). Yet, despite these theoretical advances, there is still a lack of *empirical* evidence linking reform-oriented teaching practices and improved student outcomes on measures of algebraic understanding. It is exactly this relationship that the Algebra Teaching Study seeks to explore.

In order to achieve this goal, our team is: (1) developing an observation scheme for capturing effective classroom practices in algebra classrooms, (2) altering existing measures of algebraic understanding in order to capture students' robust understanding of algebra, and (3) seeking connections between these two sets of measures. In particular, we decided to focus on students' understanding of word problems in algebra, both because of their central importance to the traditional algebra curriculum and students' well-documented struggles with them (cf. Nathan & Kim, 2007). In this paper we will focus on the development of our observation scheme; our adaption of measures of robust algebraic understanding are discussed elsewhere

(Wernet, Lepak, Seashore, Nix, & Reinholz, 2011).

Given the difficulty of establishing empirical linkages between classroom practices and positive student outcomes, we first engaged this task by focusing on word problems in eighth grade algebra classrooms. We believe that if we are successful in this endeavor it will provide the basis for establishing such linkages between classroom practices and mathematical understanding in mathematics classrooms more generally. Thus, in contrast with existing observation schemes, the scheme we are developing—the Algebra Classroom Teaching Instrument for Observing Norms (ACTION)—provides a high-resolution focus on classroom practices around the particular conceptual content of algebra. The reason for focusing on algebra is due to its role as a critical gatekeeper to college-preparatory mathematics courses, with word problems being a well-documented area of student struggle (Moses & Cobb, 2001; Reed, 1999; Schoenfeld, 2008; Verschaffel, Greer, & De Corte, 2000).

Despite a push for a reform-oriented approach to mathematics instruction, there is no consensus on which types of understandings are most important in algebra (Schoenfeld, 2004). Moreover, which classroom practices are considered "effective" is also a value judgment. Thus, in developing the ACTION scheme, we are creating a tool that embodies our values both about what we think is important in algebra, as well as what we consider to be effective algebra classroom practices. Yet, if we are successful in our enterprise, then the empirical link between these valued classroom practices and measures of student performance should provide an objective basis for these values. The establishment of such an empirical link should also have important implications for professional development, and with some modifications we see this as a potential use of the ACTION scheme.

Background: Defining a Robust Understanding of Algebra

Our team chose to focus on algebra word problems because we believed that a measure of students' proficiency with word problems would be indicative of their general proficiency with algebra. As expressed by the NCTM 2000 standards, students who are proficient with algebra should be able to: (1) understand patterns, relations, and functions; (2) represent and analyze mathematical situations and structures using algebraic symbols; (3) use mathematical models to represent and understand quantitative relationships; and (4) analyze change in various contexts (NCTM, 2000). This focus on functions, relationships, and multiple representations is prominent in the research literature (cf. Callis, Chazan, Hodges, & Schnepp, 2008; Kaput, 2000; Schwartz & Yerushalmy, 1992). Drawing from the literature, we developed a set of five criteria for a robust understanding of algebra (see figure 1; the full criteria are elaborated in appendix A).

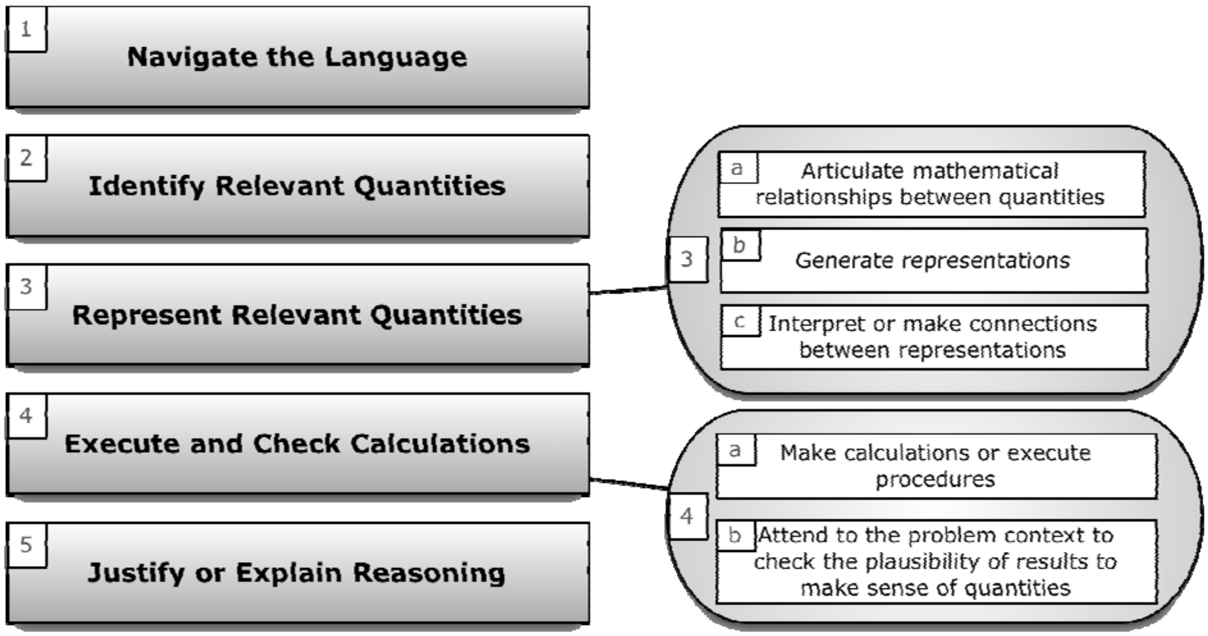

FIGURE 1. *Five Criteria for a Robust Understanding of Algebra*

However, we soon realized that a focus on "word problems" was insufficient, because many word problems do not require that students possess a robust understanding of algebra as embodied by the above criteria in order to solve them. Ultimately, instead of focusing on "word problems," we have decided to focus on what we call Contextual Algebraic Tasks (CATs) (see Wernet, et al., 2011 for more information). There is no strict set of criteria for classifying such tasks, but in general, these are tasks in which: (1) the context is both relevant and accessible, (2) students are required to navigate language, (3) algebraic reasoning is appropriate and useful for solving the task, and (4) aspects of the problem are non-routine. From this perspective, we are interested in classroom practices that lead to a robust understanding of algebra, as embodied by our robustness criteria, which will be assessed by looking at student performance on CATs.

Methods: Developing the ACTION Scheme

Building on the work of other research teams developing general tools for classroom observation, we analyzed a number of existing observation schemes, including Systematic Classroom Analysis Notation (SCAN) (Burkhardt, Beeby, & Caddy, 1980); Mathematical Quality of Instruction (MQI) (Learning Mathematics for Teaching, 2005); IQA (Junker et al., 2005); CLASS (Pianta, Hamre, Haynes, Mintz, & Paro, 2006) and Performance Assessment for California Teachers (PACT) (Darling-Hammond, Pecheone, Merino, Chung Wei, & Andree, 2002). However, because of their focus on general instructional quality, none of the extant schemes contain the specificity required to make assertions regarding the development of robust understanding of *algebra*.

Following our analysis of extant schemes, we began the development of the ACTION scheme by first selecting videotapes of classroom instruction that we felt exemplified rich classroom practice. We analyzed the tapes using a grounded approach (Glaser & Strauss, 1967), attempting to capture the "effective practices" displayed in the tapes in our scheme. However, the level of specificity in our initial schemes was better suited to an in-depth analysis of each classroom episode, and did not facilitate real-time coding. Because our aim is for the ACTION scheme to be (eventually) used for large-scale evaluations of many classrooms, this made our initial schemes inappropriate for our project's goals. Ultimately, we realized that many aspects of the IQA made it appropriate for the types of lessons we are interested in. In recent iterations of our scheme, we have made an effort to build from IQA where possible, supplementing our own unique components where IQA was not sufficient for our goals.

In particular, the unique focus of ACTION is on understanding Contextual Algebraic Tasks. In order to capture specific classroom practices leading to robust understandings of algebra, we realized that we needed to understand more explicitly the algebraic resources underlying a robust understanding of algebra. This is a current aim of our scheme development.

Results and Analyses: The ACTION Scheme

Our team is close to having completed the third iteration of the ACTION scheme. In its current incarnation, there are two major components to the scheme: (1) a list of events of interest (EoIs) used to guide and supplement a narrative account of the lesson, and (2) a set of rubrics for actually assigning scores to a lesson after the observation has taken place. In order to use the ACTION scheme, an observer takes time-stamped notes reflecting the activities taking place in the classroom, as well as statements made by students and the teacher (see figure 2).

***In-Lesson* Narrative Description**		
Time Stamp	**Narrative Description**	**EoI codes***

FIGURE 2. *In-lesson narrative account*

The note-taking process is guided by the list of events of interest (EoI codes) in the scheme. For instance, one of the events of interest in the ACTION scheme is: “Teacher presses for accuracy or asks students to provide evidence for claims.” If an observer witnesses this event of interest, he or she would note the time at which it occurred, make a note that this specific event occurred, and then provide a brief qualitative description of the event, such as actually writing down what the teacher said (e.g. “could you explain *why* you decided to factor both sides of the equation?”). Once an observer has noted all events of interest in a lesson, he or she has the objective basis with which to assign scores in the rubrics.

All rubrics within the ACTION scheme are rated on 0-4 scales. The ACTION scheme consists of 3 sets of rubrics, which operate at 3 levels specificity: (1) general classroom practices, (2) mathematical classroom practices, and (3) CAT-specific classroom practices (see figure 3). Accordingly, depending on the type of lesson being observed, how well the rubrics apply may vary. For example, the general classroom practices portion of ACTION could just as well be applied to an English classroom as a mathematics classroom, but the CAT-specific portion would only apply specifically to algebra classrooms in which a teacher is attempting to help students develop a robust understanding of algebra.

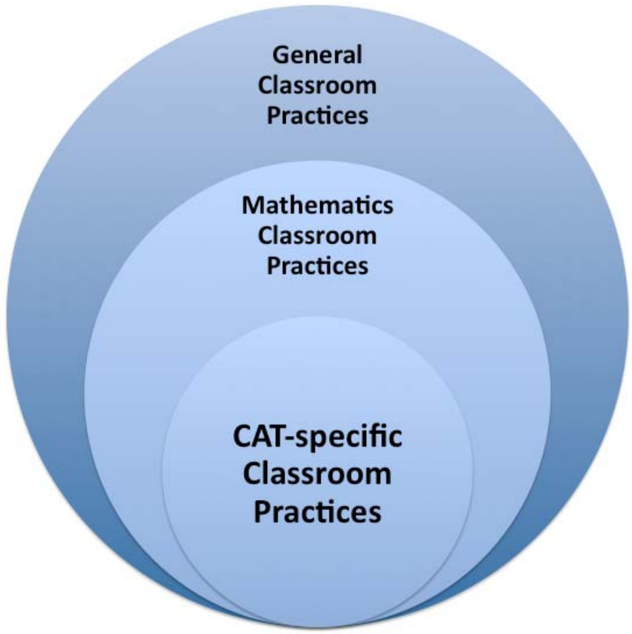

FIGURE 3. *The three levels of focus of ACTION*

Like the IQA, the rubrics in ACTION are based on quantitative tabulations of qualitative events, not subjective qualitative judgments. For instance, consider the rubric *Mathematical Accuracy* from the mathematical classroom practices portion of ACTION (see Figure 4).

Description	Low: 0	1	2	3	High: 4
Mathematical Accuracy	EVENTS OF INTEREST: (2) Teacher makes a significant mathematical error (e.g. an arithmetic error that is not recognized quickly and messes up future calculations, misusing or misrepresenting mathematical terms, getting stuck in the middle of a problem)				
	Three or more occurrences of an EoI are observed. OR teacher errors significantly compromise the lesson.	**Two** occurrences of an EoI are observed.	**One** occurrence of an EOI is observed OR teacher makes numerous minor errors.	The only mistakes made by the teacher are minor and don't detract from the lesson.	Teacher's mathematics is flawless.

FIGURE 4. *Teacher's Mathematical Accuracy Rubric*

Notice that the scorings in the rubric use actual quantifications of the event(s) of interest (i.e., how many significant mathematical errors did the teacher make?), as opposed to subjective phrasings such as, "the teacher's mathematical accuracy was high/moderate/poor." The problem with such phrasings is that they provide no objective basis for making an evaluation, since operationalizing terms like "high," "moderate," and "poor" is left to the observer.

Based on our preliminary trials, the ACTION scheme shows promise for use with real-time coding. Our team has tested ACTION with both videotapes of algebra instruction, as well as with real-time observations of algebra classrooms. We have had success in creating a narrative account of lessons in real time, with minimal time required after a lesson to fill out the rubrics. We believe near real-time coding will be feasible for an individual trained in the final version of the scheme.

An important issue that arises for creating rubrics based on quantifications of events of interest is determining what the appropriate quantities are (i.e., how many times should we expect an effective teacher to press for student reasoning during a lesson?). Our team is in the process of using the ACTION scheme on a number of videos of different algebra classrooms in order to refine the rubrics and determine the appropriate values.

See figure 5 for an example of one of the rubrics from the general classroom practices portion of ACTION. Unlike the mathematical accuracy rubric, this rubric consists of a number of different events of interest. In general, a single rubric may contain any number of events of interest, depending on how many different types of events are relevant to that dimension of classroom practice. In this rubric the number of times each EoI occurs is not counted; each EoI must simply occur at least once.

<table>
<tr><th>Description</th><th>Low: 0</th><th>1</th><th>2</th><th>3</th><th>High: 4</th></tr>
<tr><td rowspan="2">Explicit Process Requirements</td><td colspan="5">EVENTS OF INTEREST:
(4) Teacher explicitly specifies what the end product should be.
(5) Teacher provides guidelines on how students should organize themselves to work on the task.
(6) Teacher specifies amount of time allotted to work on task.</td></tr>
<tr><td>No EoIs were observed.</td><td>One or more EoIs were observed, but were only directed toward individual students or small groups.</td><td>One of the EoIs was observed.</td><td>Two of the EoIs were observed.</td><td>All three EoIs were observed.</td></tr>
</table>

FIGURE 5. *Explicit Process Requirements Rubric*

The CAT-specific classroom practices are grounded specifically in our notion of robust understanding of algebra. Figure 6 gives an example rubric from this portion of the scheme.

<table>
<tr><th>Description</th><th>Low: 0</th><th>1</th><th>2</th><th>3</th><th>High: 4</th></tr>
<tr><td rowspan="2">Identifying Relevant Quantities

EoI: 4, 5</td><td colspan="5">EVENTS OF INTEREST:
(4) Teacher calls students' attention to quantities relevant to solving the problem.
(5) Teacher calls students' attention to what the problem is asking.</td></tr>
<tr><td>Teacher does not support students in identifying relevant quantities. i.e., no EOIs are observed.</td><td>One occurrence of an EoI is observed.</td><td>Two occurrences of an EoI are observed.</td><td>Three occurrences of an EoI are observed.</td><td>Four or more occurrences of an EoI are observed.</td></tr>
</table>

FIGURE 6. *Identifying Relevant Quantities Rubric*

In this rubric scores are tabulated based on the total number of occurrences of either of the EoIs. Thus, it is possible a teacher could score a 4 on this rubric even though one of the EoIs never takes place in the classroom (although we feel this would be unlikely).

Conclusion

The development of the ACTION scheme is part of our effort to investigate an empirical relationship between "effective" reform-oriented mathematics classroom practices and students' robust understanding of algebra. The scheme operates at three different levels: (1) general classroom practices, (2) mathematical classroom practices, and (3) classroom practices related to developing a robust understanding of Contextual Algebraic Tasks (CATs). To use ACTION, an observer takes a narrative account of the lesson guided by a list of events of interest associated with each of the above categories, and uses the counts of events of interest to fill out a number of corresponding rubrics after the lesson is completed. These rubrics are rated on 0-4 scales.

Based on preliminary trials, ACTION shows promise for coding in near real-time, which will enable researchers to analyze many videos and, in the long run, perform large-scale evaluations. Further, ACTION may also be appropriated and adapted by professional developers performing classroom observations. Ultimately, unlike existing schemes, ACTION provides the types of measures required to perform a quantitative analysis of classroom practices that have been hypothesized to lead to a robust understanding of algebra.

Acknowledgements

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APPENDIX A: ROBUSTNESS CRITERIA

1. Students are able to navigate the language in a problem statement in order to make sense of the problem situation.

2. Students are able to identify which quantities are relevant to the problem situations.

3. Students are able to represent relevant quantities mathematically.

    3a. Students are able to articulate the mathematical relationships between quantities.

    3b. Students are able to generate appropriate mathematical representations.

    3c. Students are able to interpret and make connections between representations.

4. Students are able to execute procedures/calculations and check the plausibility of their results.

    4a. Students are able to execute procedures/calculations and check their results with regard to the plausibility of the mathematical operations performed.

    4b. Students attend to the problem context to check the plausibility of their results and make sense of quantities.

5. Students are able to clearly and thoroughly explain and justify their reasoning.