Assessing What Counts

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Effective algebra instruction should result in students' robust understanding of algebra. The Algebra Teaching Study seeks to make the connection between effective pedagogical moves and student learning. This paper first discusses the way that we have defined and operationalized robust understanding in defining our "robustness criteria". These criteria provide a framework for our selection of tasks that prompt students to demonstrate their understanding of algebra. Secondly, this paper discusses the processes we took in selecting tasks, assembling pre- and posttests, and revising and editing the chosen tasks. We conclude with our dilemmas in creating rubrics to score individual student work, attempting to give scores based on the robustness criteria to individual students and to classes, our unit of analysis.

The purpose of the Algebra Teaching Study (ATS) is to provide tools that can empirically support the identification of instructional practices leading to students' robust understanding of algebra. One crucial aspect of this undertaking is to measure changes in students' algebraic mathematical understanding over time and link these changes to instructional practices. To measure student understanding, we focus on word problems in algebra because fluency in solving such demands a wide range of mathematical skills and understandings, including sense making, modeling, representational and procedural skills—in short, robust understanding. In this paper, we will describe what we mean by *robust algebraic understanding,* and how we have chosen and adapted tasks to assess students' understanding. Then, we discuss how our assessments are used in the Algebra Teaching Study.

Robust Algebraic Understanding

To assess students' robust understanding using contextual algebraic tasks, we first established criteria for robust understanding. The following *robustness criteria* reflect what some literature highlights as important abilities for students to have when using algebra to solve complex algebraic word problems (Driscoll, 1999; National Council of Teachers of Mathematics, 2000; Verschaffel, Greer, & de Corte, 2000). Robust understanding means that students are able to:

1. Navigate the language to make sense of the problem situation
2. Identify relevant quantities
3. Represent relevant quantities
    a. Articulate mathematical relationships

    b. Generate appropriate algebraic representations
    c. Interpret or make connections between representations
4. Execute and check calculations
    a. Make calculations and execute procedures accurately
    b. Attend to the problem context in make sense of their results
5. Explain and justify their reasoning about the problem situation and their solution strategies.

The assessment tasks for ATS must provide opportunities for students to demonstrate these aspects robust understanding.

Contextual Algebraic Tasks

An important part of our work has been to identify the types of problems that we wish to use for assessment and observed being implemented in algebra classrooms. There are strong cultural perceptions about what constitutes mathematics word problems and how they should be solved (Thomas & Gerofky, 1997; Gerofsky, 2004). Our focus is on the development of students' robust understanding of algebra, rather than on their ability to decode which algorithm should be applied through the use of key words and ignoring "cover stories." Consequently, many traditional word problems are inadequate for our purposes because they neither require nor contribute to students' robust understanding. Consider a typical textbook problem:

> If boxes of cookies cost \$7 each, how many boxes can you buy if you have \$147? (a) Write an equation for this situation. (b) Solve your equation using guess and check.

Although this problem uses the context of buying cookies, this context is merely a cover-story; it is in no way relevant to solving the problem. Moreover, while part (a) of the problem requires students to generate an equation, in part (b) the solution is simply found using guess and check, which could have been done without generating the equation at all. Irrespective of the value we place on problems of this sort, it is clear that such a problem does not give students a sufficient opportunity to display a robust understanding of algebra as embodied by our robustness criteria.

Thus, we explicitly distinguish *contextual algebraic tasks* (CATs) from the canonical word problems that are typically seen in textbooks at the end each set of exercises (Gerofsky, 2004). We hope that by focusing on the subset of word problems we define as CATs, we can capture the development of students' sense making and robust understanding. As an exemplar CAT, consider the problem below adapted from the Connected Mathematics Project (2006):

> Santa Monica beach has a rectangular swimming zone for toddlers. One side of the swimming zone is the shore. The other three sides are formed by a 32-foot long rope, with 4 wooden posts on the corners.
>
> (1) Draw and label a picture or diagram to represent this situation. Let the length of the side parallel to the beach be *x*.

(2) How should the posts and rope be arranged to create the swimming zone of the *largest possible* area? How do you know you found the largest possible area?

(3) Write an equation for the area of the swimming zone in terms of x (the length of the side parallel to the beach).

This task illustrates the following essential characteristics of Contextual Algebraic Tasks:

- **Relevant and accessible context:** The context should be both accessible to students and appropriate for the mathematical ideas presented. CATs require students to situate their work in the context provided, as opposed to the context merely serving as a "cover story."
- **Navigation of language:** Students are required to navigate language to build a situation model and solve the problem.
- **Algebraic reasoning**: For ATS, we focus on tasks in pre-algebra or algebra that require algebraic reasoning; reasoning that goes beyond the successful execution of a known algorithm. For instance, this may mean that a CAT requires that students use algebraic representations (sometimes multiple representations) as an effective way to solve the problem.
- **Non-routine or non-algorithmic:** CATs require more than the application of previously learned calculations and procedures.

Developing Assessments from CATs

The 16 tasks (see Appendix A) we selected for our assessments were adapted from a range of sources: the Mathematics Assessment Resource Service (MARS) assessments (Shell Centre, accessible at http://www.noycefdn.org/mars2009.php), the Connected Mathematics Project curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006), the California Standards Tests Released Items (California Department of Education, 2010), and Drexel University's The Math Forum (http://mathforum.org/). In selecting the tasks for our assessments, we considered the four characteristics of CATs described above. We narrowed the content focus to key topics in algebra: linear functions, quadratic functions, and systems of equations. We choose tasks compatible with a functions-based approach to algebra (Chazan, 2000). In particular, the CATs we choose require or promote the interpretation or generation of representations and prompt students to explain their reasoning. For the context, we addressed the diversity of our populations by developing tasks students could relate to, which prompted significant changes in the wording from their original form. Likewise, focusing on functions and their representations helped address the range of curricula students were using in our sample and the fact that what might be considered routine for one group may not be for another.

The adapted tasks were parsed into four assessments with four items each (see Table 1). We purposely placed linear function tasks at the beginning of the assessments to increase student access, since students likely would have been exposed to linear functions in 7th grade. Together these assessments cover our range of content areas and robustness criteria. This design enables us to create a picture of student understanding at the classroom level, while assessing individual students on a reasonable number of tasks.

TABLE 1.

Task sets and their algebraic focus.

		Tasks Set A1	Tasks Set B1
Focus	*Linear Pattern*	Tables	Hexagons
	Linear Equation with Real-World Context	Bowling	Journey
	Systems of Equations Real-World Context	Tickets	Cart
	Quadratic with Real World Context	Swimming	Height of the Ball
		Tasks Set A2	**Tasks Set B2**
Focus	*Linear Equation with Real World Context*	Bicycles	Saving Money
	Graphs of Functional Relations	Graph Matching A	Graph Matching B
	Linear/Quadratic Pattern	Square	Tile Company
	Systems of Equations Real World Context	Carnival Tokens	Car Wash

Use and Modification of Assessments

We piloted the four versions of the assessments during the spring of 2010 with approximately 40 students (CA: n = 28, MI: n = 11) from 7 schools using either think-aloud protocols or post hoc interviews. The interviews allowed us to probe deeper into students' algebraic reasoning in order to gauge the strengths and limitations of the tasks as written. These interviews informed revisions of the assessments to better solicit students' reasoning and engage students in activities more closely aligned with the robustness criteria. For instance, in the Tile Company pattern task, we added the question, "if you are given 4 blue, 40 red, and 60 yellow squares, what is the largest square you could make that follows this pattern?" This allowed us to assess students' ability to navigate the language in the task, make sense of their equation, and use it to solve the problem.

We also reassembled the 16 tasks into four different assessments with four tasks each, shown in Table 1. In part, this was a conscious effort to match pairs of assessments for pre and post tests. That is, we attempted to pair similar tasks targeting the same algebraic content and similar robustness criteria on a pre- and post-test in order to measure individual student growth in understanding. However, the focus on individual student growth can also be problematic because the complex tasks take a considerably long time to solve, meaning students can solve at most three or four tasks in a class period.

Thus, another reason to split the tasks was to have two pretests and two posttests that can be given in each class so that in every class, all 16 tasks are covered. Our primary unit of analysis will be the class, not the individual student, so we will be looking at the trends in scores across the class. This allows for broader coverage of the topics and a better idea of what students are learning across the class. It is an assessment strategy that sacrifices what we can learn about individual students, but with this sacrifice we are afforded the opportunity to more broadly assess the content we had targeted. This approach has been used in other large-scale research projects (e.g., NAEP).

We began the 2010-2011 school year by giving pretests to students in our 10 observation classrooms, where we are assessing about 260 students (CA: $n \approx 140$, MI: $n \approx 120$) and interviewing an average of four students from each class. These interviews afforded us better access to student thinking and will be helpful in understanding links between the student work and student understanding. The information provided by the interviews will inform future modification of the assessment tasks.

Current Status

We are in the process of developing rubrics to analyze student work from the CATs. The primary goal is for these rubrics to provide feedback regarding how student work aligns with our robustness criteria. We are attempting to build the rubrics to take into account students' intermediate work and reflect the strategies that they are using rather than just their final solution. Knowing that we want to assign a score reflecting students' algebraic understanding according to each robustness criteria, we are struggling with how to subdivide the robustness criteria into finer-grained objectives that align with parts of each task while striking a balance between breadth, depth, and manageability.

Although a highly detailed examination of a small subset of student work has promised interesting trends, we have decided at this point to keep the grain size of the rubrics fairly coarse to reflect student understanding at the level of our robustness criteria. In creating the rubrics, however, we are taking note of these interesting trends to consider them further to determine if we can isolate a list of Algebraic Resources (ARs) required to solve such demanding tasks. We have not reached consensus regarding how fine-grained these ARs should be, and recognize advantages for fine-grained assessments that capture student understanding at a deep level.

To date, we have created rubrics for the tasks in two assessments: A1 and B1. In these rubrics, we have outlined possible strategies that students might use in solving the task, and delineated the robustness criteria for each strategy. For example, in tasks that ask students to create a general equation that models a situation, one possible strategy (for Carla's Cart) is to set up and evaluate numerical expressions for two constraints. In this case, the following robustness criteria would be demonstrated by and credited to the student:

RC2: Represent relevant quantities
RC3a: Articulate mathematical relationships
RC4a: Make calculations and execute procedures accurately
RC4b: Attend to the problem context in make sense of their results

Our next step is to evaluate a subset of student work using the rubrics and make adjustments as necessary.

Conclusion

One challenge ATS has faced is to develop assessments that provide information about a mathematics class's growth in robust understanding over the course of a year. The work to this point has involved adapting tasks taken from established assessments, organizing them into matched pre- and post-tests, and making decisions about how to score student work to reflect the robustness criteria. The assessments will play an important role in the ATS project as we attempt to identify instructional practices that lead to students' robust algebraic understanding by providing a means to measure growth in students' understanding across the class.

Acknowledgements

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REFERENCES

Brown, D. E. (1994). Facilitating conceptual change using analogies and explanatory medels. *16*(2), 201-214.

California Department of Education, 2010. *California Standards Test Release Items for Algebra,* available at http://www.cde.ca.gov/ta/tg/sr/css05rtq.asp.

Chazan, D. (2000). *Beyond formulas in mathematics and teaching: Dynamics of the high school algebra classroom*. New York, NY: Teachers College Press.

Driscoll, M. (1999). *Fostering algebraic thinking*. Portsmouth, NH: Heinemann.

Gerofsky, S. (2004). *A man left Albuquerque heading east: word problems as genre in mathematics education.* New York: Die Deutsche Bibliothek.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2006). *Growing, Growing, Growing* (2 ed.). Boston, MA: Pearson.

National Council of Teachers of Mathematics (2000). *Principles and standards for teaching mathematics*. Reston, CA: Author.

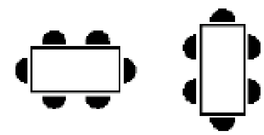
Verschaffel, L., Greer, B., & de Corte, E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets and Zeitlinger.

Appendix A

Pattern Tasks (4):

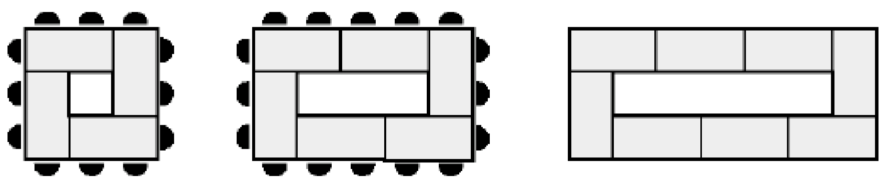
Arranging Tables
A company supplies tables for business meetings.

Each table is a rectangle, and can seat one person on its short edge, and two people on its long edge, like the figures on the right.



The diagrams below show how these tables can be made into arrangements for different numbers of people. The different arrangements are numbered, like the figures below.

No one sits inside the arrangements of tables.



Size 1
4 tables
12 people

Size 2
6 tables
16 people

Size 3

(1) How many people can sit at a Size 3 arrangement?

(2) How many people can sit around a Size 13 arrangement? Please explain.

(3) Write an equation for the number of people who can sit at a Size *S* arrangement. Explain how you know your equation will work.

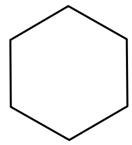
(4) What size arrangement is needed for 75 people? Explain how you know your answer is right.

Hexagons

Maria has some hexagonal tiles. Each side of a tile measures 1 inch. She arranges the tiles in rows; then she finds the perimeter of each arrangement.

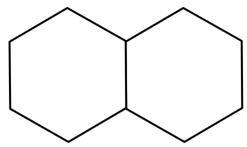
1 tile

Perimeter = 6 inches

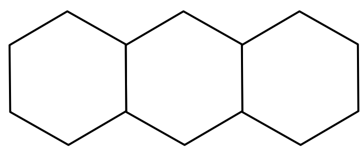


2 tiles

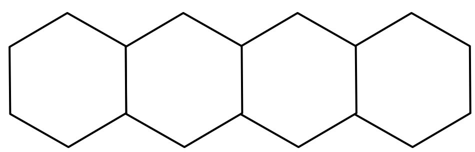
Perimeter = 10 inches



3 tiles



4 tiles



(1) Find the perimeter of her arrangement of 4 tiles.

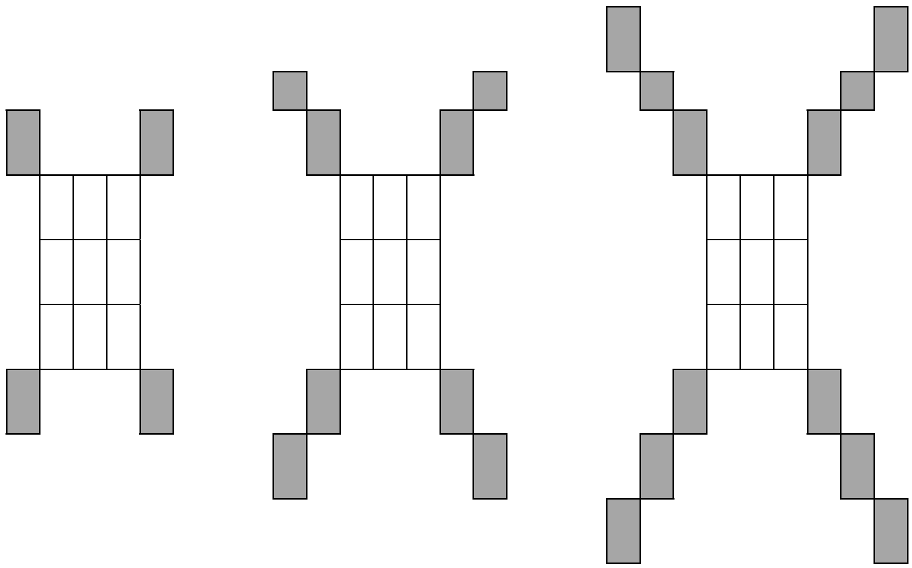
(2) What is the perimeter of a row of 10 tiles? Explain how you figured it out.

(3) Write an equation for finding the perimeter of a row of hexagonal tiles when you know the number of tiles, *n*, in the row.

(4) Maria made a long row of hexagon tiles. She made a small mistake when counting the perimeter and got 67 inches. How many tiles do you think were in her row? Explain why you think this.

Square Patterns
Mary has some white and gray square tiles.

She uses them to make a series of patterns like these:



Pattern #1 Pattern #2 Pattern #3

(1) How many gray tiles will be used in Pattern #4? How many total?

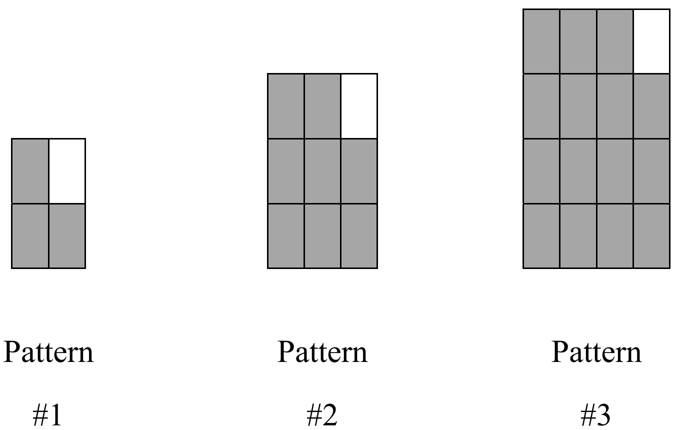
(2) What is the total number of tiles she needs to make Pattern #10? Explain how you figured it out. (Can you do it without drawing?)

Mary uses 93 tiles in all to make one of the patterns.

(3) What is the number of the pattern she makes? Show your work.

(4) Write a formula for finding the total number of tiles Mary needs to make pattern # n.

Mary now uses gray and white square tiles to make a different pattern.

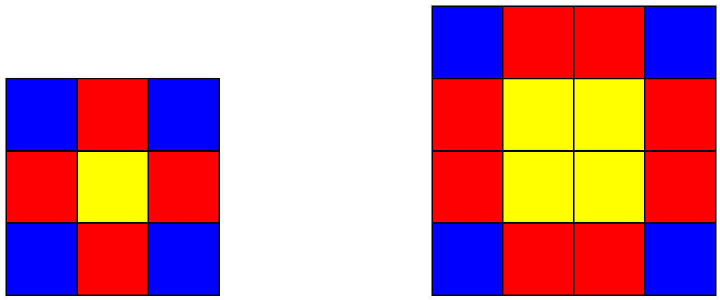


(5) How many gray tiles will there be in Pattern #10? Explain how you figured it out.

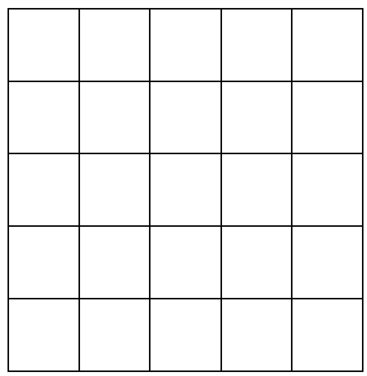
(6) Write an algebraic formula linking the pattern number, P, with the number of gray tiles, T.

(7) What is the largest square pattern Mary could make if she was given 175 grey square tiles?

Tile Company
The Terryton Tile Company makes floor tiles. One tile design uses grids of small, colored squares as shown below in the 3x3 and 4x4 patterns.



(1) Fill in the 5x5 design, indicating which tiles are blue (B), red (R), and yellow (Y).



(2) Suppose you apply the same design rule to a 10x10 grid. How many small squares will be blue? Yellow? Red?

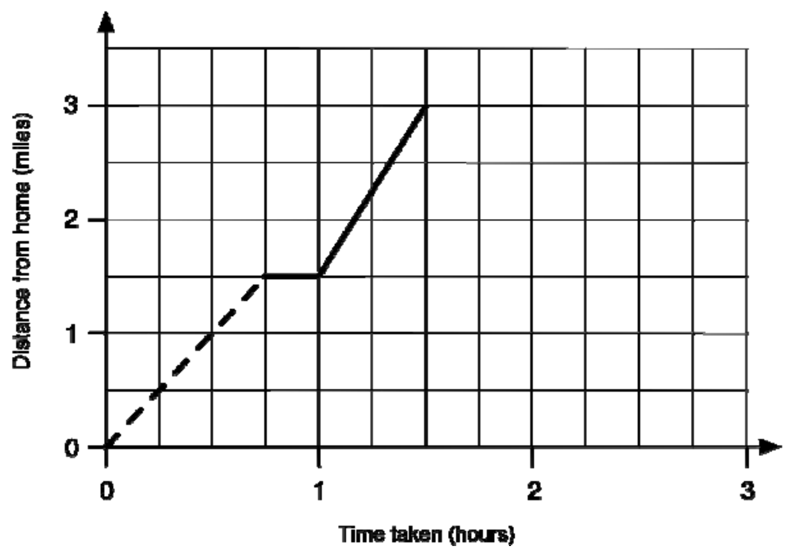
(3) How many small squares of each color will there be if you apply the rule to an *n*x*n* grid? Please explain your answer.

(4) If you are given 4 blue, 40 red, and 60 yellow squares, what is the largest square you could make that follows this pattern?

Linear-"Real-World" (4)

Going Bowling

Craig and James walked from their house to the bowling alley during their summer vacation. The graph below shows their trip to the bowling alley, which was three miles away. They left home at 10:30am.



(1) What do you think they did after they traveled 1.5 miles? How do you know?

(2) At what speed did they walk for the first part of the trip (shown on the graph with a dotted line)? Show how you figured it out.

(3) After an hour, the graph becomes steeper. What does this tell you about what Craig and James did?

Craig and James bowled for an hour and then took a bus home. The bus averaged 12 miles per hour.

(4) What time did they get home? Show your work.

(5) Explain how you figured out your answer for (4).

(6) Continue the graph on the previous page to show the rest of this information.

Journey

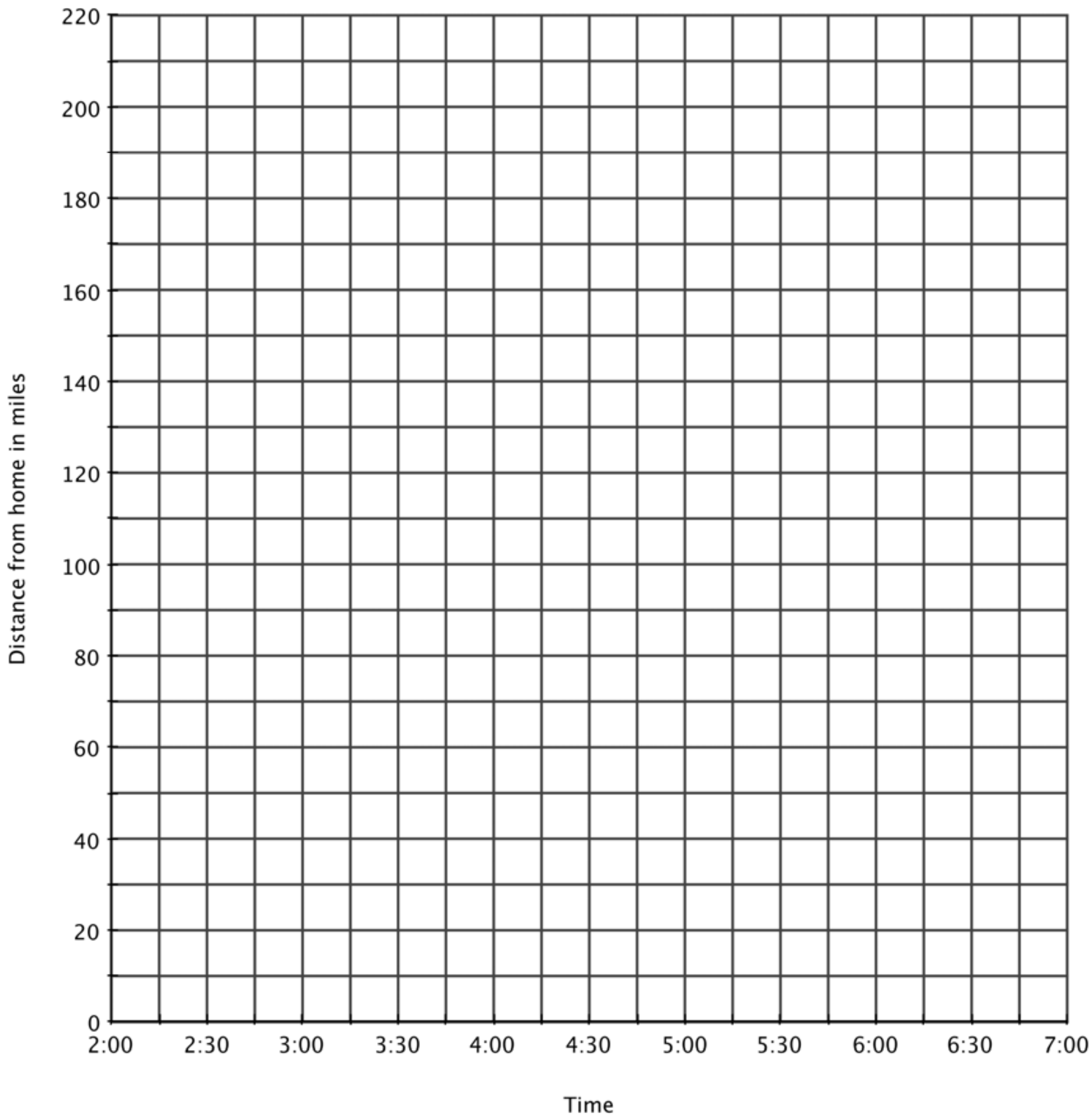
Here is a description of a car journey:

> "I left home at 2:00pm. I traveled for *half an hour* at forty miles an hour, then for an *hour* at fifty miles an hour. I had a *half hour* stop for lunch. Then I traveled another 110 miles in the last *2 hours*."

(1) Complete this table showing the distances traveled by the end of each stage of my journey.

Times in hours	2:00	2:30	3:30	4:00	6:00
Distance from home in miles	0				

(2) Draw a distance-time graph for this journey on the grid below.



(3) How fast was I traveling between 5 and 5:30 pm? Show your work.

(4) What was my average speed for the whole journey? Show your work.

(5) At what time was I 60 miles from home? Explain how you know.

(6) If I turned around at 6:00 and drove home, traveling 40 miles per hour, what time would I get home?

(7) Explain how you figured out your answer to (6).

Renting Bicycles

The table below shows the cost of renting a bicycle at Joe's Bike Shop. To rent a bicycle, you have to pay Joe a fee to adjust the bicycle to your size. You also pay for each hour that you rent the bicycle.

Hours	Cost (in dollars)
2	20
5	35
8	50

(1) How much does it cost to rent a bicycle for 3 hours? Show how you know.

(2) José rented a bicycle from Joe for 24 hours. How much should it have cost him? Show your work.

(3) Write an equation for the cost, *C*, of renting a bicycle for *H* hours.

(4) Explain what each number in your equation means.

Saving Money

Here is some information about how much money Carla, Alex, Sue, and Ben have during the school year.

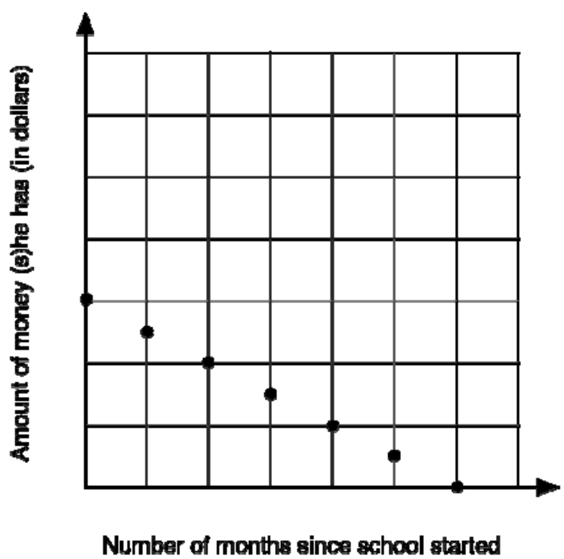
Carla: Her mom gave her $150 at the beginning of the school year, and Carla saved $25 every month after that.

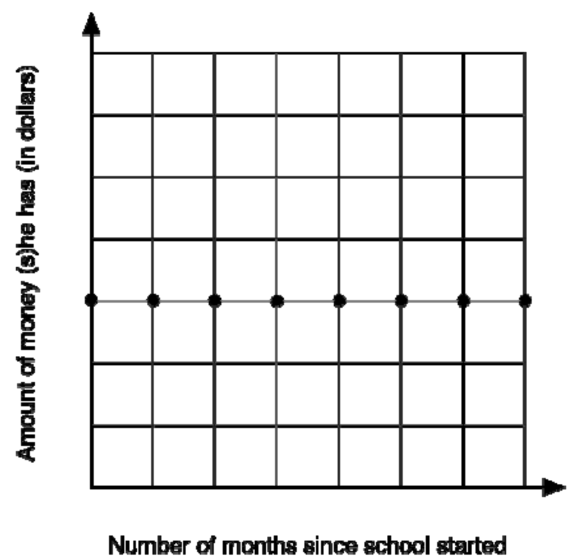
Alex: Alex put $150 in his piggy bank when school started and never added or took out money.

Sue: At the beginning of the school year, Sue had $150 in her piggy bank. After that, she paid her cousin $25 each month to buy his bike from him.

Ben: Ben started with nothing, but he saved $25 every month.

(1) Here are some graphs illustrating these situations. Match each person with a graph and explain how you decided.

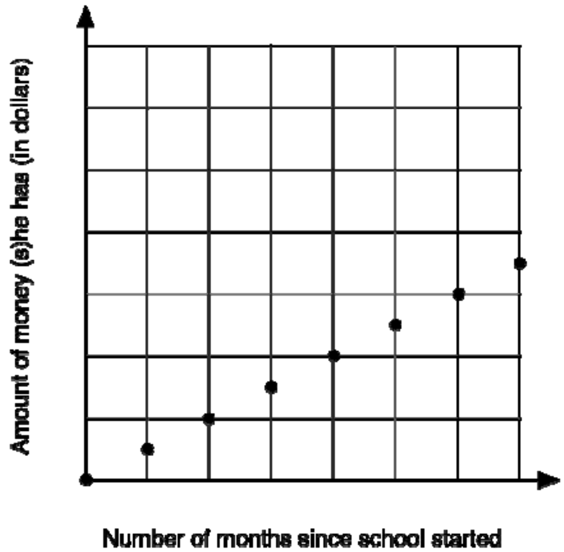


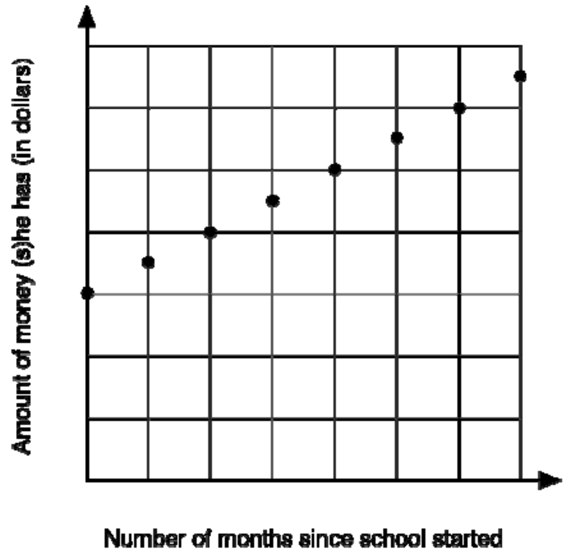


Name: ______________________________ Name: ______________________________

Reason: _____________________________ Reason: _____________________________

____________________________________ ____________________________________





Name: ______________________________ Name: ______________________________

Reason: ____________________________ Reason: ____________________________

____________________________________ ____________________________________

(2) Add numbers to each graph to show the scale on the x- and y-axis of each graph.

(3) In these equations, A is the amount of money each student has at different times through the year and n is the number of months since school started.

$A = 150 - 25n$ $\quad$ $A = 150n + 25$

$A = 150n$ $\quad$ $A = 25n + 150$

$A = 150$

For each person, find an equation that represents the amount of money that he or she had at different times through the year. If none of the equations above match, write your own.

Carla: ______________________________ Sue: ______________________________

Alex: ______________________________ Ben: ______________________________

(4) The amount of money that Carla's friend David has through the school year is represented by this equation:

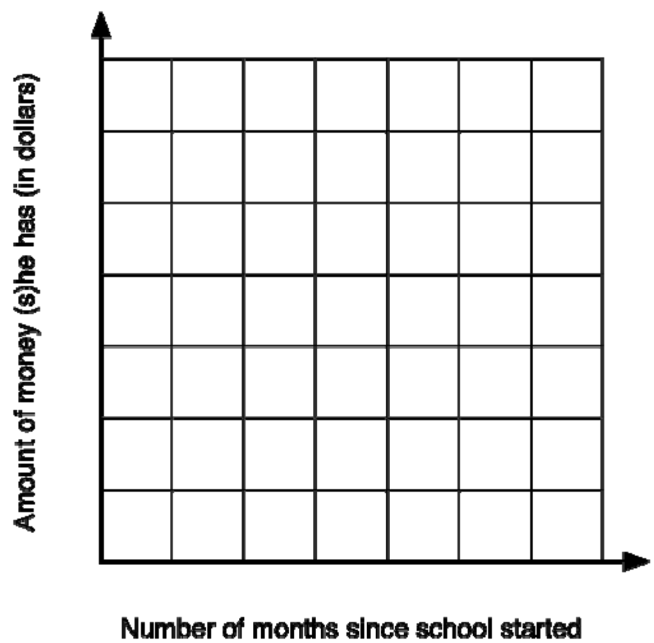
$A = 50n + 150$

(A is the amount of money and n is the number of months since school started.)

Write a story about what David could have been doing, and draw the matching graph below.

____

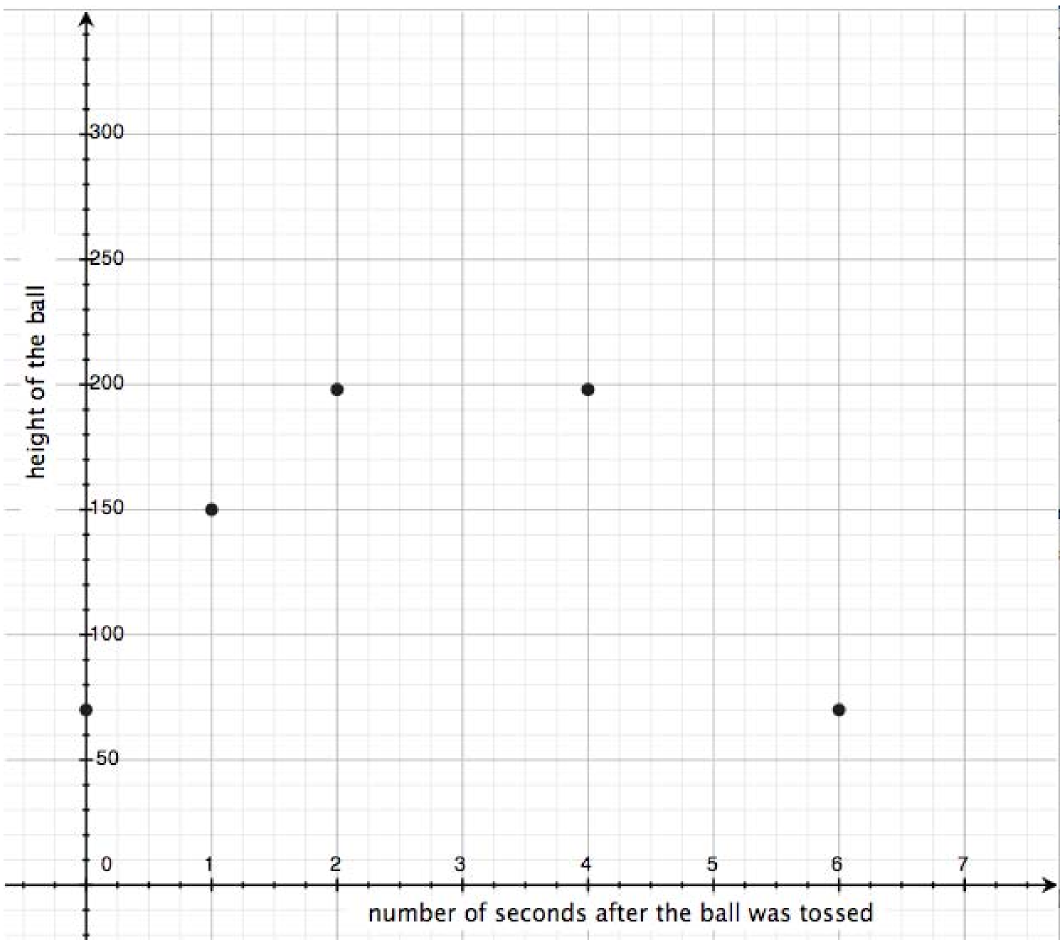
____



Height of the Ball

In algebra class, Mary learned a formula for the height of an object that is tossed in the air: $y = -16x^2 + v\ x + d$. In this formula, *y* is the height of the object *x seconds* after it thrown, *d* is the starting height of the object and *v* is the speed (in feet per second) that the object is moving when it is first thrown.

In a problem that Mary's teacher gave her, a ball was thrown off the top of a 70 foot tower. Mary started working on the problem by creating the graph below.



(1) How high was the ball 6 seconds after it was thrown?

(2) How high was the ball 5 seconds after it was thrown?

(3) What is the highest the ball will go? When will it be at this height? Explain how you know.

(4) Use the letter M to label the maximum height of the ball on the graph.

(5) When will the ball hit the ground?

(6) Tell how accurate you think your answers to parts (2) – (5) are. What could make your answers more accurate?

(7) Write an equation for the height of the ball, y, after x seconds. Show all your work.

(8) What was the speed of the ball when it was first thrown? Explain how you found your answer.

Safe Zone for Swimming

Santa Monica beach has a rectangular swimming zone for toddlers. One side of the swimming zone is the shore. The other three sides are formed by a 32-foot long rope, with 4 wooden posts on the corners.

(1) Draw and label a picture or diagram to represent this situation. Let the length of the side parallel to the beach be x.

(2) How should the posts and rope be arranged to create the swimming zone of the *largest possible* area? How do you know you found the largest possible area?

(3) Write an equation for the area of the swimming zone in terms of x (the length of the side parallel to the beach).

Systems of Equations (4)

Carla's Cart

Carla's family is throwing a huge party. Her mom gives Carla the rolling cart and says, "I want you to take this to the store to get boxes of crackers and bottles of punch for the party. Get as many items as you can with $45.00. Make sure you bring back any change."

Carla says, "Sure, I'll do that. But how many of each do you want me to get?"

Her mom replies, "Just get as many items as you can, but make sure not to overload the cart. I don't want you pushing more than 30 pounds. "

Carla does some algebra as she walks to the store. Bottles of punch weigh about 3 pounds, and cost $1.50 each. Boxes of crackers weigh about 1 pound, and cost $2.00 each. Based on this information, she decides how many of each item to get.

(1) Find a combination of crackers and punch that Carla could purchase. Show that your answer is possible.

(2) Could Carla buy 6 bottles of punch and 10 boxes of crackers? Explain how you know.

(3) Carla purchased the largest number of items she could bring home. How many bottles of juice and boxes of crackers did she buy? Explain how you know your answer is right.

(4) How much change will Carla bring her mom? Show how you figured it out.

Printing Tickets

Carmen is organizing the printing of tickets for a show.

She has collected prices from several copy centers and the two displayed below seem to be the best:

Best Tix
Best Tix
Tickets printed
\$10 to set up
plus
\$1 for 20 tickets

Sure Print
SURE PRINT
TICKET PRINTING
20 TICKETS FOR \$2

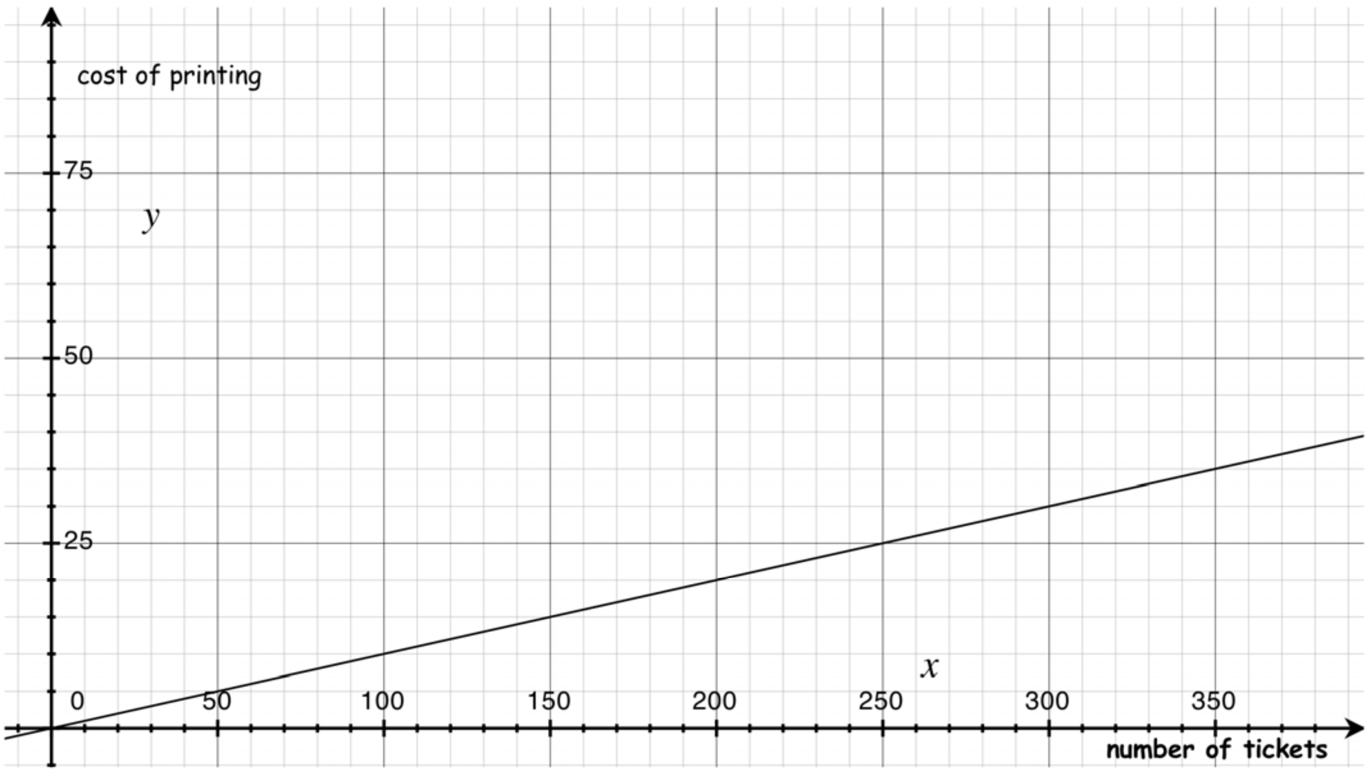
Using y for the cost of printing, and x for the number of tickets, Carmen writes a formula for each of the copy centers. This is her formula for *Sure Print:*

$$y = \frac{2x}{20}$$

(1) Write a formula for *Best Tix*:

(2) Explain how you figured out the formula for *Best Tix:*

(3) Carmen's brother Rob has drawn a graph showing the cost vs. number of tickets at *Sure Print.* Draw the graph for *Best Tix* on the same graph.



(4) Carmen used the equations to find the values of *x* and *y* when the cost of printing tickets is the same for both of the copy centers. Rob used the graphs and got the same answer. What values of *x* and *y* did they get? Explain and show how each of them may have found their values of *x* and *y*.

Equations	**Graphs**

(5) What do Rob's graphs and Carmen's calculations tell you about the cost of the tickets?

Which company should Carmen choose under what circumstances?

Carnival Tokens

Alicia has total of 100 blue and red tokens from the carnival. The blue tokens are worth \$5. The red tokens are worth \$2. The total value of the tokens is \$317.

(1) How many red tokens does Alicia have? Show all your calculations to answer this question. If you use a calculator, write down what you type into it, to record what you are doing.

(2) Use words to explain how you found your answer.

(3) Jamie said, "I solved it! X equals 122." Explain why you agree OR disagree with Jamie.

Car Wash

Members of a senior class held a car wash to raise funds for their senior prom. They charged \$3 to wash a car and \$5 to wash a pick-up truck or a sport utility vehicle. They earned a total of \$450 by washing a total of 132 vehicles.

(1) How many trucks did they wash? Show all your calculations to answer this question. If you use a calculator, write down what you type into it, to record what you are doing.

(2) Use words to explain how you found your answer.

(3) Maria said, "I solved it! X equals 315." If you disagree, explain to Maria where she went wrong. If you agree, explain why.

Graph Matching (2)

Graph Matching-A

From the graphs labeled A-L (on the page after the questions), choose one that best fits each situation below.

At the Olympics, a world champion body builder held a heavy barbell over his head for a few unsteady moments, and then, with a violent crash, he dropped it. Find a graph that shows the relationship between the height of the barbell and how much time has passed.

(1) Label the axes on the graph you chose.

(2) Explain why you chose that graph:

When I ride my bike, I start slowly, gradually get faster until I'm riding at a comfortable speed, and then gradually slow as I near the end of my ride. Find a graph that shows the relationship between the distance I have traveled and how much time has passed.

(3) Label the axes on the graph you chose.

(4) Label the part on the graph that shows me riding at "a comfortable speed." How do you know?

(5) Sketch a graph that would show me coming to a sudden stop rather than a gradual one.

Explain why you drew it this way.

At the bowling alley, you pay a fee to rent shoes, and then an additional charge for each game you play. Find a graph that shows the relationship between the amount you pay and the number of games you play.

(6) Label the axes on the graph you chose.

(7) If the bowling alley increased their prices for each game (without changing the fee to rent shoes), how would the graph change?

Graph Matching-B

From the graphs labeled A-L (on the page after the questions), choose one that best fits each situation below.

When I first learned to swim, I got better very quickly. But I have found that over time, it becomes harder and harder to keep improving. Find the graph that shows the relationship between my skill level and time.

(1) Label the axes on the graph you chose.

(2) Label the place on the graph that shows me getting better very quickly. How do you know?

(3) How do you know which axis is "my skill level" and which axis is "time"? Can the labels be switched?

My new cell phone plan charges a monthly fee that includes unlimited talking. It does not include texting, though. Every time I send a text, I am charged an additional fee. Find a graph that shows the relationship between my total monthly cell phone bill and the number texts that I send.

(4) Label the axes on the graph you chose.

(5) If the phone company increased their prices for each text, how would the graph change?

If I want to change the shape of a rectangle, but not change its area, I could decrease the length and increase the height so their product (the area) stays the same. Find the graph that shows the relationship between the length and height of rectangles if their areas are all the same. Label the axes length and height.

(6) Label the axes on the graph you chose.

(7) Explain why you chose this graph.